Series Estimations

Integral Test

Consider the series $\sum_{n=k}^{\infty} a_n$ and a function $f(x)$ so that we have

- $a_n \geq 0$ For Every n
- $f(n) = a_n$ For Every n
- $\{a_n\}$ (or if you have established the last dot $f(x)$) is decreasing

then we know for and $t \geq k$

$$s_t + \int_{t+1}^{\infty} f(x)\, dx \ \leq s \leq s_t + \int_{t}^{\infty} f(x)\, dx$$

where $s = \sum_{n=k}^{\infty} a_n$ and $s_t = \sum_{n=k}^{t} (-1)^n a_n$.

Alternating Series Test

Consider the series $\sum_{n=k}^{\infty} (-1)^n a_n$ such that following hold:

- $a_n \geq 0$ FOR EVERY n
- $\lim_{n\to\infty} a_n = 0$
- $a_n \geq a_{n+1}$ FOR EVERY n

Then $\sum_{n=k}^{\infty} (-1)^n a_n$ converges!
and we can estimate with the following error for $t \geq k$

$$|s - s_t| \leq a_{t+1}$$

where $s = \sum_{n=k}^{\infty} (-1)^n a_n$ and $s_t = \sum_{n=k}^{t} (-1)^n a_n$.

Problem 1. Use the Integral Test and $n = 3$ to estimate the value of $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$

Problem 2. Use the alternating series test and $n = 16$ to estimate the value of $\displaystyle\sum_{n=2}^{\infty} \frac{(1)^n n}{n^2+1}$

Problem 3. Use the Integral Test and $n = 8$ to estimate the value of $\sum_{n=2}^{\infty} \frac{1}{n\left(\ln(n)\right)^2}$

Problem 4. Use the Integral Test and $n = 14$ to estimate the value of $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^{\frac{5}{2}}}$.

Problem 5. Use the Alternating Series Test and $n = 12$ to estimate the value of $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$.

Problem 6. Use the Alternating Series Test and $n = 18$ to estimate the value of $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{3n+4}$.