Limits Prep Worksheet 1

Basics of Exponents

Positive Whole Number Exponents

Below when I write a I mean some number, or some function, *seriously like* sin *or* x^2+3x+7, and when I write n I mean a whole number you know like 1,2,3,... Also when I use dots I mean "and so on"

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n}$$

That is *iterated* multiplication.

Example(s):

$$2^3 = 2 \cdot 2 \cdot 2$$
$$(x^2-7x+2)^4 = (x^2-7x+2) \cdot (x^2-7x+2) \cdot (x^2-7x+2) \cdot (x^2-7x+2)$$

Negative Whole Number Exponents

Still when I write a I mean some number that is **not zero**, or some function that is **zero for every** x, *seriously like* $\sin(x)$ *or* x^2+3x+7, and when I write n I mean a whole number you know like 1,2,3,... Also when I use dots I mean "and so on"

$$a^{-n} = (\ldots(((1 \div \underbrace{a) \div a) \div a) \div \ldots \div a)}_{n} = \underbrace{\frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} \cdot \ldots \cdot \frac{1}{a}}_{n}$$

That is *iterated* division.

Example(s):

$$2^{-1} = \frac{1}{2}$$
$$4^{-3} = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{4^3}$$
$$(x^2-2x+1)^{-2} = \frac{1}{(x^2-2x+1)} \cdot \frac{1}{(x^2-2x+1)} = \frac{1}{(x^2-2x+1)^2}$$

Fractions as Exponents

Below when I write a I mean some number, or some function, *seriously like* sin *or* x^2+3x+7, and when I write n I mean a I mean a whole number you know like 1,2,3,... Also when I use dots I mean "and so on"

$$a^{\frac{1}{n}} = \sqrt[n]{a} = b \quad \text{when ever} \quad b^n = a$$

That is the n^{th} root.

Example(s):

$$4^{\frac{1}{2}} = \sqrt{4} = 2$$
$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$
$$(x^2+2x+1)^{\frac{1}{2}} = \sqrt{x^2+2x+1} = (x+1)$$

Basic Properties

Still when I write a or b I mean some number that is **not zero**, or some function that is **zero for every** x, *seriously like* $\sin(x)$ *or* x^2+3x+7, and when I write n or m I mean a whole number you know like 1,2,3,... Also when I use dots I mean "and so on"

$$a^0 = 1 \qquad a^n \cdot a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m} \qquad (a^n)^m = a^{n\cdot m}$$

$$\left(a \cdot b\right)^n = a^n \cdot b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Problem 1. Try to write each of these expressions using as few exponents as possible, trying to only use positive exponents. Assume that all variables represent nonzero real numbers.

1. $(-2)^2 3^3$
2. $\left(\frac{2}{5}\right)^2$
3. $-(3^2 \cdot 5)^0$
4. 30^0
5. -30^0
6. $(-30)^0$
7. $20^0 + 30^0$
8. $\left(x^3 y^{-7} k^5 4\right)^0$
9. 4^{-2}
10. 3^{-3}
11. $(2x)^{-2}$
12. $-5r^{-2}$
13. $3^{-3} + 9^{-2}$
14. $3^{-1} - 6^{-1}$
15. $\left(\frac{1}{5}\right)^{-2}$
16. $\left(\frac{2}{3}\right)^{-3}$
17. $\left(\frac{3z}{4}\right)^{-3}$
18. $\left(\frac{1}{5x}\right)^{-2}$
19. $\frac{r^9}{r^{12}}$
20. $\frac{k^7}{k^3}$
21. $\frac{c^7}{b^3}$
22. $\frac{12^{-7}}{12^{-6}}$

23. $\dfrac{3}{3^{-4}}$

27. $\left(\dfrac{-2y^3}{z^4}\right)^6$

30. $\dfrac{x^7y}{(x^2)^3}$ (assuming $x \neq 0$)

24. $\dfrac{a^5}{b^3}$

28. $3^3 \cdot 3^n$

25. $\left(\dfrac{-4x}{5}\right)^3$

31. $(x^2+y)^3(x^2+y)^2$

26. $\left(3x^4\right)^2$

29. $\dfrac{2^a3^a}{6^b}$

32. $\left(3a^{-2}b^{-4}\right)$

33. $\dfrac{12w^7w^{-3}}{20w^{-1}w^5}$

34. $\left(-5r^{-2}s^5t^{-3}\right)^2\left(sr^2s^{-3}t\right)^{-2}$

35. $\dfrac{9^{-3}}{9^79^{-2}}$

36. $\left(-2p^2\right)(3q)^0\left(5r^2\right)$

37. $\dfrac{3^{-2}x^{-4}\left(x^2\right)^{-3}}{2\left(x^2\right)^{-1}}$

38. $\left(3x^2y^{-2}\right)^{-2}\left(2x^{-2}y\right)^{-3}$

39. $\left(\dfrac{a^6b^{-2}}{2a^{-2}}\right)^{-1}\cdot\left(\dfrac{6a^{-2}}{5b^{-4}}\right)^2\cdot\left(\dfrac{2b^{-1}a^2}{3b^{-2}}\right)^{-1}$

WARNING: "The Freshmen's Dream"

BE VERY CAREFUL NOT TO MAKE THIS COMMON MISTAKE!!!!

$$\left(a+b\right)^n \neq a^n + b^n$$

Problem 2. Try to write each of these expressions using as few exponents as possible, trying to only use positive exponents. Assume that all variables represent nonzero real numbers.

1. $-\sqrt{121}$
2. $\sqrt[3]{216}$
3. $\sqrt[3]{-125}$
4. $-\sqrt[3]{512}$
5. $\sqrt{(-10)^2}$
6. $\sqrt[8]{-1}$
7. $\sqrt{\dfrac{64}{81}}$
8. $\sqrt[4]{\dfrac{81}{16}}$
9. $\left(\sqrt{-10}\right)^2$
10. $\sqrt[3]{m^9}$
11. $-\sqrt[3]{-27}$
12. $\sqrt{12^2}$
13. $-\sqrt[4]{16}$
14. $-\sqrt[5]{\dfrac{1}{32}}$
15. $\sqrt[4]{k^{20}}$
16. $121^{1/2}$
17. $16^{1/4}$
18. $125^{1/3}$
19. $(-32)^{1/5}$
20. $(64)^{3/2}$
21. $-32^{3/5}$
22. $-16^{5/2}$
23. $(-8)^{3/2}$
24. $27^{-4/3}$
25. $\left(\dfrac{64}{125}\right)^{-2/3}$
26. $8^{3/4}$
27. $(9q)^{5/8} - (2x)^{2/3}$

28. $(5y)^{-3/5}$

29. $(2y+x)^{2/3}$

30. $\sqrt{2^{12}}$

31. $\sqrt[3]{4^9}$

32. $\sqrt[3]{y}\sqrt{y}$

33. $\dfrac{\sqrt[3]{t^4}}{\sqrt[5]{t^4}}$

34. $3^{1/2} \cdot 3^{3/2}$

35. $\dfrac{64^{5/3}}{64^{4/3}}$

36. $r^{-8/9} \cdot r^{17/9}$

37. $\dfrac{k^{1/3}}{k^{2/3} \cdot k^{-1}}$

38. $\dfrac{(p^3)^{1/4}}{(p^{5/4})^2}$

39. $p^{2/3}(p^{1/3} + 2p^{4/3})$

40. $\dfrac{\sqrt[3]{k^5}}{\sqrt[3]{k^7}}$

41. $\sqrt[3]{xz} \cdot \sqrt{z}$