Chain Rule

Differentiation Rules

Let $y = f(x)$ and $y = g(x)$ be functions which are differentiable at x. Let a and b be constants.

Linearity:

$$D_x\left[af(x) + bg(x)\right] = af'(x) + bg'(x)$$

Product Rule:

$$D_x\left[f(x) \cdot g(x)\right] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Quotient Rule:

In the case $g(x) \neq 0$

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left[g(x)\right]^2}$$

Chain Rule:

In the case that f is diferentiable at x and g is differentiable at $f(x)$

$$D_x\left[g\Big(f(x)\Big)\right] = g'\Big(f(x)\Big)f'(x)$$

The Functions

Students at time can become overwhelmed with all the possible functions. To help this lets list them here.

Powers

$$x^n \qquad \frac{d}{dx}x^n = nx^{n-1}$$

Trig

$$\sin(x) \qquad \frac{d}{dx}\sin(x) = \cos(x)$$

$$\cos(x) \qquad \frac{d}{dx}\cos(x) = -\sin(x)$$

$$\vdots \qquad \vdots$$

Exponential

$$e^x \qquad \frac{d}{dx}e^x = e^x$$

Logarithm

$$\ln(x) \qquad \frac{d}{dx}\ln(x) = \frac{1}{x}$$

Composition

Now that we have our list of functions in hand lets discuss the key player in the chain rule *COMPOSITION*!

$$f\Big(g(x)\Big)$$

I will call $f(x)$ the **OUTSIDE FUNCTION** and
I will call $g(x)$ the **INSIDE FUNCTION**!
Now lets look at our different functions in this different roles!

x^n: **POWERS on the OUTSIDE**

$$\Big(\dots\Big)^n$$

Examples:

$$\Big(x^2 + 2x + 7\Big)^3$$

$$\Big(\sin(x)\Big)^4 = \sin^4(x)$$

$$\Big(e^x\Big)^2 = e^{2x}$$

Trig: **Trig function on the OUTSIDE**

$$\sin\Big(\dots\Big)$$

Examples:

$$\sin\Big(x^2 + 3x + 7\Big)$$

$$\tan\Big(e^x\Big)$$

$$\sec\Big(e^x + 7x + \ln(x)\Big)$$

e^x: **Exponential on the OUTSIDE**

$$e^{\Big(\dots\Big)}$$

Examples:

$$e^{3x^2+7x+10}$$

$$e^{\sin(x)}$$

$$e^{5x+\tan(x)}$$

$\ln(x)$: **Logs on the OUTSIDE**

$$\ln\Big(\dots\Big)$$

Examples:

$$\ln\Big(x^2 + 8x - 2\Big)$$

$$\ln\Big(\sin(x) + 4\Big)$$

$$\ln\Big(e^x - 7x + 4\Big)$$

Problem 1. In this next question JUST <u>IDENTIFY</u> which function is the INSIDE function and which function is the OUTSIDE function.

i. $h(x) = \sqrt{x^2 + 2x + 7}$

iii. $h(x) = \tan\left(\sqrt[3]{3x^2 + 4} + 7\right)$

v. $h(w) = e^{w^4 - 3w^2 + 9}$

ii. $h(x) = \left(2t^3 + \cos(t)\right)^5$

iv. $h(x) = \ln\left(x^{-4} + x^7 + 2\right)$

vi. $h(t) = \sec\left(\tan(x) + x^{-7}\right)$

Problem 2. Differentiate the given functions (using the chain rule!)

i. $f(x) = \cos\left(x^2 \mathbf{e}^x\right)$

iv. $y = \sqrt[3]{1 - 8z}$

ii. $f(x) = \left(6x^2 + 7x\right)^4$

v. $f(t) = 5 + \mathbf{e}^{4t + t^7}$

iii. $g(t) = \left(4t^2 - 3t + 2\right)^{-2}$

vi. $g(x) = \mathbf{e}^{1 - \cos(x)}$

vii. $H(z) = 2^{1-6z}$

viii. $u(t) = \cos(3t - 1)$

ix. $g(y) = \ln\left(1 - 5y^2 + y^3\right)$

x. $V(x) = \ln(\sin(x) - \cos(x))$

xi. $h(z) = \sin\left(z^6\right) + \sin^6(z)$

xii. $S(w) = \sqrt{7w} + \mathbf{e}^{-w}$

xiii. $g(z) = 3z^7 - \sin\left(z^2 + 6\right)$

xiv. $f(x) = \ln(\sin(x)) - \left(x^4 - 3x\right)^{10}$

xv. $h(t) = t^6 \sqrt{5t^2 - t}$

xvi. $q(t) = t^2 \ln\left(t^5\right)$