Chapter P
Section P.6
Problem 1. Write the following rational expressions in lowest terms. If there are any restrictions on the variables, indicate them.

(a) $\dfrac{9 - x^2}{x^2 + 6x + 9} = \dfrac{(3-x)(3+x)}{(x+3)(x+3)}$

$= \dfrac{(3-x)}{(x+3)}$

$x \neq \pm 3$

(c) $\dfrac{s^2 - s - 6}{s^2 + s - 12} = \dfrac{(s-3)(s+2)}{(s-4)(s+3)}$

$s \neq 4, -3$

(b) $\dfrac{22r^2 - 11r}{6 - 12r} = \dfrac{11r(2r-1)}{6(1-2r)}$

$= \dfrac{-11r(1-2r)}{6(1-2r)}$

$= \dfrac{-11r}{6}$ $\quad r \neq \frac{1}{2}$

(d) $\dfrac{7 - b}{b - 7} = -1$

$b \neq 7$

Problem 2. Add, subtract, multiply, or divide as indicated. Write all answers in lowest terms. If there are any restrictions on the variables, indicate them.

(a) $\dfrac{n}{m + 3} - \dfrac{-3n + 7}{m + 3}$

$= \dfrac{4n - 7}{m + 3}$

$m \neq -3$

(c) $\dfrac{4x}{8x + 4} \cdot \dfrac{14x + 7}{6} = \dfrac{28x(2x+1)}{24(2x+1)}$

$= \dfrac{28x}{24} = \dfrac{7x}{6}$

$x \neq \frac{-1}{2}$

(b) $\dfrac{r}{r^2 - s^2} + \dfrac{s}{r^2 - s^2}$

$= \dfrac{(r+s)}{(r-s)(r+s)} = \dfrac{1}{r-s}$

$r \neq \pm s$

(d) $\left(y^2 - 4\right) \div \dfrac{4y + 8}{5}$

$= \dfrac{5(y-2)(y+2)}{4(y+2)} = \dfrac{5(y-2)}{4}$

$y \neq -2$

1

(e) $\dfrac{9}{2t} + \dfrac{4}{t^2}$

$\dfrac{9t+8}{2t^2}$

$t \neq 0$

(i) $\dfrac{3x+12}{6x-30} \cdot \dfrac{x^2-x-2}{x^2-16}$

$\quad 6(x-5) \qquad (x-4)(x+4)$

$\dfrac{3(x+4)(x-2)(x+1)}{6(x-5)(x-4)(x+4)}$

$= \dfrac{(x-2)(x+1)}{(x-5)(x-4)}$

$x \neq 5, 4, -4$

(f) $\dfrac{12}{p^2+6p+9} - \dfrac{2}{p+3}$

$(p+3)^2$

$\dfrac{12-2(p+3)}{(p+3)^2} = \dfrac{-2p+9}{(p+3)^2}$

$p \neq -3$

(j) $\dfrac{27-3k^2}{3k^2+8k-3} \div \dfrac{k^2-6k+9}{6k^2-19k+3}$

$\dfrac{3(9-k^2)(6k-1)(k-3)}{(3k-1)(k+3)(k-3)(k-3)}$

$k \neq \frac{1}{3}, \pm 3 \quad \dfrac{3(9-k^2)(6k-1)}{(3k-1)(k+3)(k-3)}$

$\dfrac{3(6k-1)}{(3k-1)}$

(g) $\dfrac{1}{x^2-1} - \dfrac{1}{x^2+3x+2}$

$(x-1)(x+1) \qquad (x+2)(x+1)$

$\dfrac{x+2-(x-1)}{(x-1)(x+1)(x+2)} \qquad x \neq \pm 1, -2$

$\dfrac{x+2-x+1}{(x-1)(x+1)(x+2)} = \dfrac{3}{(x-1)(x+1)(x+2)}$

(k) $\dfrac{2}{3-m} - \dfrac{2}{m-3} + \dfrac{3}{m^2-9}$

$\qquad\qquad\qquad (m-3)(m+3)$

$\dfrac{-2(m+3)-2(m+3)+3}{(m-3)(m+3)}$

$\dfrac{-2m-6-2m-6+3}{(m-3)(m+3)} \qquad m \neq \pm 3$

$\dfrac{-4m-9}{(m-3)(m+3)} = \dfrac{-(4m+9)}{(m-3)(m+3)}$

(h) $\dfrac{9k-18}{6k+12} \cdot \dfrac{3k+6}{15k-30}$

$\dfrac{9(k-2)\,3(k+2)}{6(k+2)\,15(k-2)} = \dfrac{3}{4}$

$k \neq \pm 2$

(l) $\dfrac{4z}{z^2+6z+8} - \dfrac{2z-1}{z^2-z-6}$

$(z+4)(z+2) \qquad (z-3)(z+2)$

$\dfrac{4z(z-3)-(2z-1)(z+4)}{(z+4)(z+2)(z-3)} \qquad z \neq -4, 2, 3$

$\dfrac{4z^2-12z-2z^2-8z+z+4}{(z+4)(z+2)(z-3)}$

$\dfrac{2z^2-19z+4}{(z+4)(z+2)(z-3)}$

Additional Problems

EP 1. Add, multiply, or divide as indicated. Write all answers in lowest terms. If there are any restrictions on the variables, indicate them.

(a) $\dfrac{x^2 + 10x + 21}{x^2 + 14x + 49} \cdot \dfrac{x^2 + 12x + 35}{x^2 - 6x - 27}$

$$\frac{(x+7)(x+3)(x+7)(x+5)}{(x+7)^2(x-9)(x+3)}$$

$$\frac{x+5}{x-9}$$

$$x \ne -7, 9, -3$$

(c) $\dfrac{3r + 4}{3} + \dfrac{6r + 4}{6}$

$$\frac{2(3r+4)+2(3r+2)}{6}$$

$$\frac{6r+12+6r+4}{6} = \frac{12r+16}{6}$$

$$= \frac{4(3r+4)}{6} = \frac{2(3r+4)}{3}$$

(b) $\dfrac{z^4 + 2z^3 + z^2}{z^5 - 4z^3} \div \dfrac{9z + 9}{6z + 12}$

$$\frac{z^2(z+1)^2 \, 6(z+2)}{z^3(z^2-4) \, 9(z+1)}$$

$$\frac{2(z+1)}{3z(z-2)}$$

$$z \ne 0, \pm 2, -1$$

(d) $\dfrac{2k^2 + 5k - 12}{2k^2 + k - 3} \div \dfrac{k^2 + 8k + 16}{2k^2 + 11k + 12}$

$\overset{(2k-3)(k+4)}{}$

$\underset{(2k-3)(k+1)}{}$ $(2k+3)(k+4)$

$$\frac{(2k-3)(k+4)(2k+3)(k+4)}{(k+4)(k+4)(2k-3)(k+1)} = \frac{2k+3}{k+1}$$

$$k = -4, \frac{3}{2}, -1$$

3