Chapter P
Section P.3

Problem 1. If possible, find or simplify each root.

(a) $-\sqrt{121}$ $= -11$

(b) $\sqrt[3]{216}$ $= \sqrt[3]{2^3 \cdot 3^3} = 2 \cdot 3 = 6$

(c) $\sqrt[3]{-125}$ $= -5$

(d) $-\sqrt[3]{512}$ $= -\sqrt[3]{2^8} = -(2^{8/3})$

(e) $-\sqrt[4]{16}$ $= -\sqrt[4]{2^4} = -2$

(f) $\sqrt[8]{-1}$

Doesn't exist?

(g) $\sqrt{\dfrac{64}{81}}$ $= \sqrt{\dfrac{2^6}{3^4}} = \dfrac{2^3}{3^2} = \dfrac{8}{9}$

(h) $\sqrt[4]{\dfrac{81}{16}}$ $= \sqrt[4]{\dfrac{3^4}{2^4}} = \dfrac{3}{2}$

(i) $-\sqrt[5]{\dfrac{1}{32}}$ $= -\sqrt[5]{2^{-5}} = -2^{-1}$
$= \dfrac{-1}{2}$

(j) $-\sqrt[3]{-27}$ $= -(-3) = 3$

(k) $\sqrt{12^2}$ $=$ 12

(l) $\sqrt{(-10)^2}$ $= |10|$

(m) $\sqrt[3]{m^9}$ $= m^3$

(n) $\sqrt[4]{k^{20}}$ $= \cancel{k^5}$

$*$ $\sqrt[4]{k^{20}} = |k|^5$

Problem 2. If possible, evaluate each exponential.

(a) $121^{1/2}$ $= \pm 11$

(b) $16^{1/4}$ $= \pm 2$

(c) $125^{1/3}$ $= 5$

(d) $(-32)^{1/5}$ $= -2$

(e) $(64)^{3/2}$ $= 2^9 = 512$

why parenthesis?

(f) $-32^{3/5}$ $= -8$

(g) $-16^{5/2}$ $= -4^5 = -1{,}024$

(h) $(-8)^{3/2}$ Does not exist

(i) $27^{-4/3}$ $= \dfrac{1}{3^4} = \dfrac{1}{81}$

(j) $\left(\dfrac{64}{125}\right)^{-2/3}$ $= \dfrac{5^2}{4^2} = \dfrac{25}{16}$

Problem 3. Write each exponential as a radical. Assume that all variables represent positive real numbers.

(a) $8^{3/4}$ $= \sqrt[4]{512}$

(b) $(9q)^{5/8} - (2x)^{2/3}$

$\sqrt[8]{9^5} - \sqrt[3]{4x^2}$

(c) $(5y)^{-3/5}$

$\sqrt[5]{\frac{1}{5y^3}}$

(d) $(2y + x)^{2/3}$

$\sqrt[3]{(2y+x)^2}$

Problem 4. Write each radical as an exponential. Simplify. Assume that all variables represent positive real numbers.

(a) $\sqrt{2^{12}}$ $= 2^6 = 64$

(b) $\sqrt[3]{4^9}$ $= 4^3 = 64$

(c) $\sqrt[3]{y}\sqrt{y}$ $= y^{\frac{1}{3}+\frac{1}{2}} = y^{\frac{5}{6}}$

(d) $\dfrac{\sqrt[3]{t^4}}{\sqrt[5]{t^4}}$ $= \dfrac{t^{4/3}}{t^{4/5}} = t^{4/3 - 4/5} = t^{\frac{20-12}{5}} = t^{\frac{8}{5}}$

Problem 5. Simplify each expression. Write answers in exponential form with only positive exponents. Assume that all variables represent positive real numbers.

(a) $3^{1/2} \cdot 3^{3/2}$

$= 3^{\frac{1}{2}+3/2} = 3^{4/2} = \pm 3^2$

(b) $\dfrac{64^{5/3}}{64^{4/3}}$ $= 64^{5/3-4/3} = 64^{1/3} = 4$

(c) $r^{-8/9} \cdot r^{17/9}$ $= r^{-8/9+17/9}$

$= r$

(d) $\dfrac{k^{1/3}}{k^{2/3} \cdot k^{-1}}$ $= k^{1+1/3-2/3}$

$= k^{2/3}$

(e) $\dfrac{(p^3)^{1/4}}{(p^{5/4})^2}$ $= p^{3/4-\frac{5}{2}} = p^{\frac{3-10}{2}} = p^{-\frac{7}{2}} = \dfrac{1}{p^{7/2}}$

(f) $p^{2/3}(p^{1/3} + 2p^{4/3})$

$= p^{2/3+\frac{1}{3}} + 2p^{4/3+2/3}$

$= p + 2p^2$

(g) $\dfrac{\sqrt[3]{k^5}}{\sqrt[3]{k^7}}$ $= k^{5/3-7/3} = k^{-2/3} = \dfrac{1}{k^{2/3}}$

(h) $\sqrt[3]{xz} \cdot \sqrt{z}$ $= x^{1/3} z^{\frac{1}{3}+1} = x^{1/3} z^{4/3}$