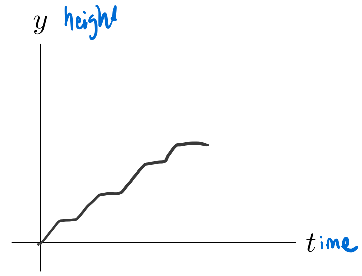
Chapter 2
Sections 2.1 and 2.2
Warm-up Problem A. At summer camp, a child comes out every morning to raise a flag. Sketch a graph of what this situation might look like if the height of the flag is on the vertical axis and time is on the horizontal axis. For the moment, do this on your own without talking to anyone.



Warm-up Problem B. Once everyone in your group has drawn their own graph, compare your work with each other. Decide which graph is the most reasonable, and send someone to draw it on the board.

Warm-up Problem C. Which of these do you think is the best graph? Why?

Problem 1. Give an example of a function encountered in everyday life. What is the input and what is the output?

time v.s. my attention level

Problem 2. Give an example of something that is not a function.

$|y| = x$

Problem 3. Determine the input and output in the following statement: "The average height in inches, h, of a young peach tree in South Carolina in a given year, is a function of r inches of cumulative rainfall since March 1."

input: r
output: height

Problem 4. Use the following table to fill in the blanks.

x	0	2	4	6	8
$f(x)$	5	6	3	0	2

a) $f(2) = \underline{6}$

b) $f(4) = \underline{3}$

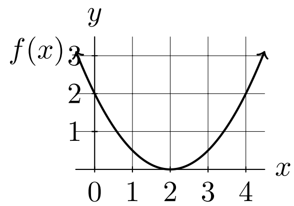
c) $f(\underline{0}) = 5$

d) $f(\underline{8}) = 2$

Is f a function? Why or why not?

yes, 1 input has unique output

Problem 5. Using the following graph, fill in the blanks.



(a) $f(4) = \underline{2}$ (b) $f(\underline{2}) = 0$

Problem 6. Using the definition of a function, explain why the Vertical Line Test correctly determines if a graph can represent a function.

a vertical line is

x = # so if it hits more than one

more than one output

Problem 7. Determine whether each of the following is a function (and of what variable). If not, what adjustments could you make so that it is a function?

(a) The number of donuts that Joe the baker can make if he has n dollars.

assuming all donuts have same cost

yes function of n

(b) $y = z^2$

function of z

(c) $y = \frac{x}{1-x}$

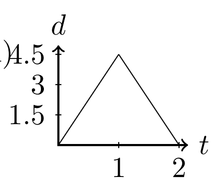
function of x

(d)

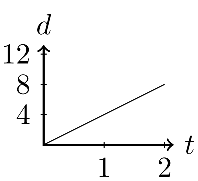
x	y
1	5
2	5
3	4
3	3
4	2

not a function

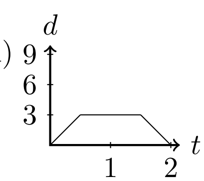
Problem 8. *Synthesis Problem*: Shannon decides to go for a run. For each of the graphs below, write a possible description of Shannon's run if t is time in hours since the start of her run, and d is her distance from home in miles.

(i)


She runs and turns around w/ litterally no change in speed

(ii)


just runs away at constant speed

(iii)


runs takes a break runs back

Problem 9. Determine whether each table below represents an increasing or a decreasing function.

(a)

t	-2	0	2	4
$h(t)$	-8	0	8	64

increasin

(b)

t	4	3	2	1
$r(t)$	5	10	15	20

decreasin

(c)

t	0	2	4	8
$s(t)$	-25	-20	-15	-10

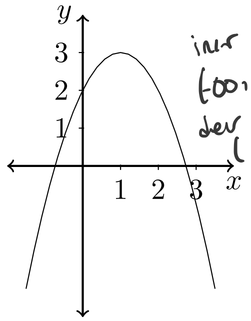
incr

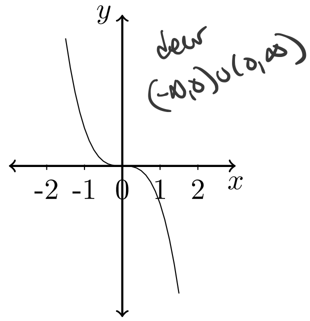
(d)

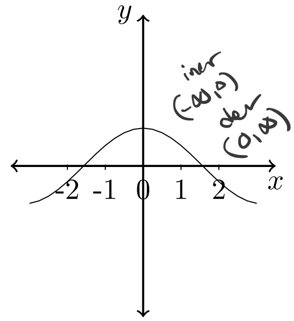
t	0	-24	-48	-72
$m(t)$	5	10	15	20

decr

Problem 10. On which intervals are the following functions increasing? Decreasing?





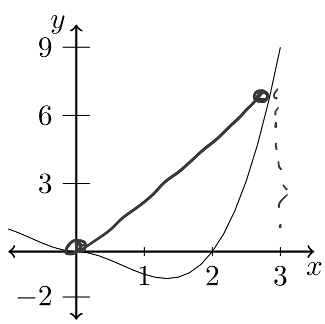


Problem 11. The distance between the sun and the horizon is a function of the time of day. Over what time interval might this be an increasing function? A decreasing function?

incr: dawn to noon

decr: noon to sunset

Problem 12. Let $g(x) = x^3 - 2x^2$. A graph of $g(x)$ is shown below. Draw a line segment between the points on the function where $x = 0$ and $x = 3$. Compute the average rate of change of $f(x)$ on the interval $[0, 3]$. What is the connection between the line segment and the average rate of change?

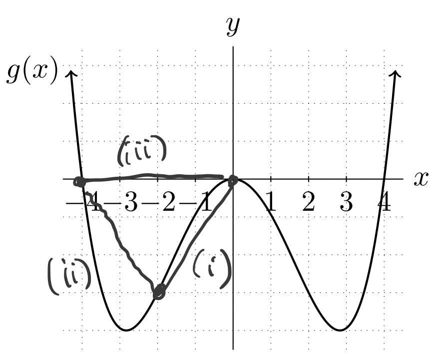


it's the slope of line drawn

$$\frac{6-0}{3-0}$$

Problem 13. Consider the function $g(x) = \frac{1}{16}x^4 - x^2$, which is graphed below.

(a) Evaluate each of the following, then interpret each as the average rate of change of $g(x)$ over some interval.



(i) $\dfrac{g(0) - g(-2)}{0 - (-2)}$

$\dfrac{0-(-3)}{2} = \dfrac{3}{2}$

(ii) $\dfrac{g(-2) - g(-4)}{-2 - (-4)}$

$\dfrac{-3-0}{2} = \dfrac{-3}{2}$

(iii) $\dfrac{g(0) - g(-4)}{0 - (-4)} = 0$

(b) For each of the expressions in (i)-(iii) of part (b), draw a line segment on your graph of $g(x)$ whose slope corresponds to the calculated rate of change.

Problem 14. Compute the average rate of change for $g(x) = 2x^2$ on the following intervals:

(a) $0 \le x \le 1$

$$\frac{g(0)-g(1)}{0-1} = \frac{0-2}{-1} = 2$$

(b) $[-1, 1]$

$$\frac{g(-1)-g(1)}{-1-1} = \frac{0}{-2} = 0$$

(c) From $x = -4$ to $x = 2$

$$\frac{g(-4)-g(2)}{-4-2} = \frac{2\cdot16-2\cdot4}{-6} = -4$$

(d) From $x = b$ to $x = a$

$$\frac{g(b)-g(a)}{b-a} = \frac{2b^2-2a^2}{b-a} = \frac{2(b-a)(b+a)}{(b-a)} = 2(b+a)$$

(e) From $x = a + h$ to $x = a$

$$\frac{g(a+h)-g(a)}{a+h-a} = 2(2a+h) = 4a+2h$$

Problem 15. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for each function below and simplify it.

(a) $f(x) = 2x$

$$\frac{2x+2h-2x}{h} = 2$$

(b) $f(x) = x^2 + 3x$

$$\frac{\cancel{x^2}+2xh+h^2+\cancel{3x}+3h-\cancel{x^2}-\cancel{3x}}{h} = 2x+3+h$$

(c) $f(x) = \sqrt{x}$

we expect?

$$\frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{\sqrt{x+h}-\sqrt{x}}{2x+h}$$

(d) $f(x) = \dfrac{2}{x}$

$$\frac{\frac{2}{x+h}-\frac{2}{x}}{h} = \frac{2x-2(x+h)}{hx(x+h)} = \frac{-2\cancel{h}}{\cancel{h}x(x+h)} = \frac{-2}{x(x+h)}$$

Problem 16. *Synthesis Problem* The amount that Mary drives changes from week to week, so the amount that she spends on gas each week also changes. However, she has calculated that, over the last year, she spent a total of \$1,450 on gas.

(a) On average, how much does she spend each week on gas?

how many weeks in a year?

$$\frac{1450}{\#\text{ of weeks}}$$

(b) If nothing major changes, how much should Mary budget for gas for the next 3 weeks?

$$3 \cdot \frac{1450}{\#\text{ of weeks}}$$

Warm-up Problem D. Find the domain and range of the function $f(x) = \sqrt{-x+4}$.

D: $-x+4 \geq 0 \Rightarrow 4 \geq x$

range: $y \geq 0$

Warm-up Problem E. Find the domain and range of the function $g(x) = \dfrac{1}{x-8}$.

D: $x-8 \neq 0 \quad x \neq 8$

R: $y \neq 0 \quad \rightarrow y = \frac{1}{x-8} \Rightarrow y(x-8) = 1 \quad 8y+1 = yx$

$yx - y8 = 1 \quad x = \frac{8y+1}{y}$

Problem 17. Let $h(x)$ be given by the formula below.

$$h(x) = \begin{cases} 4 & \text{for } x \leq 0 \\ 4 - x^2 & \text{for } 0 < x < 2 \\ 2x - 6 & \text{for } x > 2 \end{cases}.$$

(a) Find $h(-2)$, $h(0)$, $h\left(\frac{1}{2}\right)$, $h(1)$, and $h(5)$.

$h(-2) = 4 \quad h\left(\frac{1}{2}\right) = 4 - \left(\frac{1}{2}\right)^2 = 4 - \frac{1}{4}$

$h(5) = 2 \cdot 5 - 6 = 10\ 6$

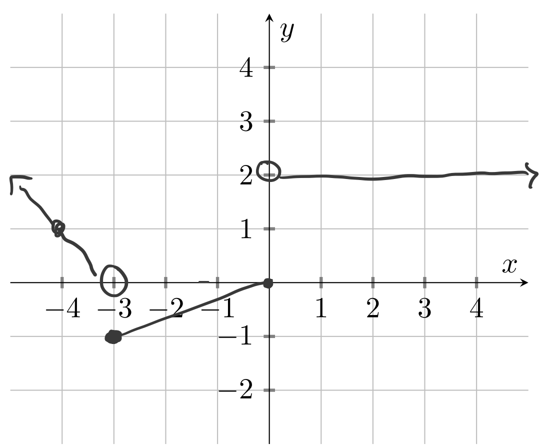
(b) What is the domain of $h(x)$?

$x \neq 2$

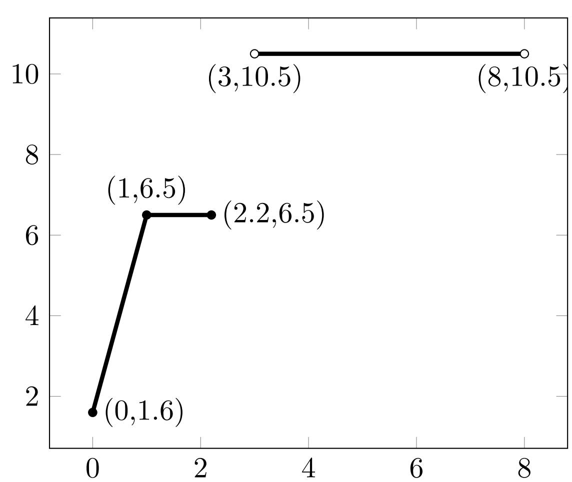
Problem 18. Graph the piecewise function $f(x)$ given below.

+3 −3

$$f(x) = \begin{cases} -x-3, & x < -3 \\ \frac{1}{3}x, & -3 \leq x \leq 0 \\ 2, & x > 0 \end{cases}.$$



Problem 19. Write a formula for the piecewise function graphed below.



$$f(x) = \begin{cases} (6.5-1.6)x - 1.6 & x \leq 1 \\ 6.5 & 1 < x \leq 2.2 \\ 10.5 & 3 < x < 8 \end{cases}$$

Problem 20. Fred has a piggy bank. On his twelfth birthday, he started saving money. Every week, on the first day of the week, he puts half of his \$20 allowance into the bank. Write a formula for $f(t)$, the amount of money in his piggy bank t days after his twelfth birthday, on the domain $0 \leq t \leq 28$. You may assume that Fred's first deposit was made on his birthday, i.e. on day $t = 0$.

weeks

$\frac{20}{2} = 10$ $\quad$ $\frac{10}{7}$ a day

* seriously
why are word problems
written like this

$$f(t) = \frac{10}{7} \cdot t + 10$$

Problem 21. An art professor wishes to take his class on a trip to the Art Institute of Chicago. The Art Institute has several different pricing options for groups:

- \$22 per person for group sizes up to 14 people (this is the standard individual rate);
- \$20 per person for group sizes from 15 to 25 people;
- \$18 per person for group sizes of 26 people or more.

For groups of at least 15 people, the group must also book their tickets ahead of time, and pay a flat reservation fee of \$10. For groups of size 14 people or fewer, there is no reservation fee.

(a) Make a table showing the cost of admittance, $C(n)$, for a group of n people, for n from 0 to 30, in increments of 5 people.

n	0	5	10	15	20	25	30
$C(n)$	0	$22 \cdot 5$	$22 \cdot 5$	$20 \cdot 5 + 10$	$20 \cdot 20 + 10$	$25 \cdot 20 + 10$	$18 \cdot 30 + 10$

(b) Write a piecewise function for $C(n)$.

$$C(n) = \begin{cases} 22 \cdot n & 0 \leq n \leq 14 \\ 20 \cdot n + 10 & 15 \leq n \leq 25 \\ 18 \cdot n + 10 & 26 \leq n \end{cases}$$

(c) How much will it cost to buy tickets for the professor plus 18 students?

$$C(18) = 18 \cdot 20 + 10 = 370$$

(d) If the professor's budget is \$220, how many students can he take (that is, how many people in addition to himself)?

$$20 \cdot n + 10 = 220$$

$$20 \cdot n = 210$$

$$n = \frac{210}{20} = \frac{21 \cdot 10}{2 \cdot 10} = \frac{21}{2}$$

$20 \cdot 11 + 10 = 230$

$20 \cdot 10 + 10 = 210$

no more than
9 extra people!

Additional Problems

EP 1. **Focus Problem** Yellow star thistle is an invasive plant species; it is a big threat in the Santa Monica Mountains national recreation area. Ecologists have gathered data to model the spreading of the plant. Suppose the land area (in square miles) that has been overtaken by the thistle t years after 2010 is given by

$$w(t) = 0.792t^3 - 4.196t^2 + 3.872t + 24.021.$$

Compute the average rate of change of the land inhabited by yellow star thistle over the following intervals of years:

(a) 2010 to 2012

$\Downarrow$ $t=0$ $\Downarrow$ $t=2$

$$\frac{w(2)-w(0)}{2-0} = \frac{21.317-24.021}{2} = \frac{-2.704}{2} = -1.352$$

(b) 2012 to 2014

$\Downarrow$ $t=2$ $\Downarrow$ $t=4$

$$\frac{w(4)-w(2)}{4-2} = \frac{23.061-21.317}{2} = \frac{1.744}{2} = 0.872$$

(c) 2014 to 2016

$\Downarrow$ $t=4$ $\Downarrow$ $t=6$

$$\frac{w(6)-w(4)}{6-4} = \frac{67.269-23.061}{2} = \frac{44.208}{2} = 22.104$$

What does this suggest about the future of the park?

the spread is speeding up

EP 2. A section of highway has a 55 mph speed limit. If you are caught speeding between 56 and 74 mph, your fine is \$50 plus \$3 for every mile over 55 mph. For 75 mph and higher, your fine is \$150 plus \$5 for every mile per hour over 75.

(a) How much would you be fined if you were caught traveling

(i) 60 mph? $50+(3)(5) = 65$

(ii) 90 mph? $150+(5)(7) = 185$

(b) What is the difference of the speeding ticket cost if you are caught going 74 mph versus 75 mph?

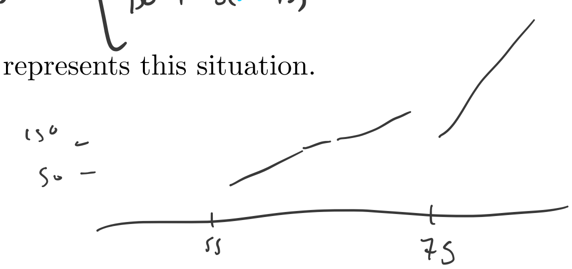
$74 \rightarrow 50+(3)(17) = 50+51 = 101$ $\quad 75 \rightarrow 150+5(1) = 155$

$155-101 = 54$

(c) Find a function $C = f(s)$ where C represents the cost of the ticket and s represents speed.

$$f(s) = \begin{cases} 50 + 3(s-55) & 56 \le s \le 74 \\ 150 + 5(s-75) & 75 \le s \end{cases}$$

(d) Sketch a graph that represents this situation.



(e) What are the domain and range of the cost function?

D: $(55, \infty)$

R: $(50, 101) \cup (155, \infty)$

EP 3. Graph the following:

$$f(x) = \begin{cases} 2 & \text{for } x < 1 \\ x+1 & \text{for } x \geq 1 \end{cases}.$$

Where is the graph increasing? Decreasing? Constant?

incrs grtr t 1

nev decr

con smaller th 1

EP 4. Sketch a graph of the following:

$$g(x) = \begin{cases} x^2 - x & \text{for } -5 \leq x < -2 \\ 4 - 4x & \text{for } -2 \leq x \leq 5 \end{cases}.$$

Determine the domain and range, as well as where the function is increasing and where it is decreasing.

D: $(-5, 5)$

R: $(-16, 30)$