



Getting to Know Your 500-Level MATH Courses at the University of South Carolina

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MATH 511: Probability (3 Credits)

Probability and independence; discrete and continuous random variables; joint, marginal, and conditional densities, moment generating functions; laws of large numbers; binomial, Poisson, gamma, univariate, and bivariate normal distributions.

Prerequisite or Corequisite: C or better in MATH 241.

Cross-listed course: STAT 511

The purpose of this course is to give you an introduction to probablity theory and probablity distributions. The material presented will not only serve as a basis for the subsequent courses, STAT 512/513, but is also extremely useful and fascinating in its own right. STAT 511 has a prerequisite of a standard multivariable calculus course, and a strong familiarity with differentitation, integration, infinite series and sequences, and related facts, is necessary. This course is very important for those of you considering careers in actuarial sciences.

The course covers the axiomatic approach to probability, counting techniques, Bayes Theorem, random variables, probability distributions for discrete and continuous random variables, mathematical expectation, moment generating functions, joint and conditional distributions for multiple random variables, and measures of association (covariance and correlation). This course focuses on both theory and application. You will be expected to derive theoretical results using algebra and calculus and apply these results to problems from a multitude of applications.



George Androulakis

MATH 515 (STAT 523): Financial Mathematics II (3 Credits)

Convex sets. Separating Hyperplane Theorem. Fundamental Theorem of Asset Pricing. Risk and expected return. Minimum variance portfolios. Capital Asset Pricing Model. Martingales and options pricing. Optimization models and dynamic programming.

Prerequisite: Grade of C or better in MATH 514 (STAT 522)

Cross-listed course: STAT 523

Complex mathematical techniques are now widely used in finance. This course is the second part of a two course sequence that provides an elementary introduction to these techniques and to the fundamental concepts of financial mathematics. We examine some mathematical models that are used in finance, e.g. to model risk and return of financial assets and to model the random nature of stock prices.

In the first part of the course we continue our investigation of the 'No Arbitrage Principle'. A very important mathematical result known as the Separating Hyperplane Theorem will be proved in order to establish the 'Fundamental Theorem of Asset Pricing' and the closely related 'Arbitrage Theorem'. We also consider some other interesting mathematical consequences of the Separating Hyperplane Theorem such as the existence of a steady state vector for Markov matrices. In the second part of the course we look at some mathematical problems arising in portfolio selection. Taking into account the investor's tolerance for risk leads to the problem of maximizing expected 'utility'. Using the technique of 'meanvariance analysis' we derive the main results of the famous Capital Asset Pricing Model which relates the riskiness of an individual stock to the overall market through a numerical coefficient known as 'beta'. If time permits we will consider some discrete optimization problems such as the 'knapsack problem' which arise in portfolio selection. In the third part of the course we shall return to the problem of options pricing. Using the multi-period binomial model we shall find an algorithm for the pricing of the American put option. The existence of an 'equivalent martingale measure' for which the discounted stock process is a 'fair game' leads us naturally to the general theory of martingales. We shall examine some of the basic concepts and results of this theory such as the notion of a 'stopping time' and Doob's Optional Sampling Theorem.



Lili Ju

MATH 520: Ordinary Differential Equations (3 Credits)

Differential equations of the first order, linear systems of ordinary differential equations, elementary qualitative properties of nonlinear systems.

Prerequisites: C or better in MATH 344 or MATH 544.

Upon completion of this course, students will be knowledgeable about and will be able to analyze solutions to differential equations of the first order and linear systems of ordinary differential equations. They will also be able to apply these ideas to determine elementary qualitative properties of nonlinear systems.

Differential equations is the language of science. Many basic scientific laws express the change in one quantity in terms of the values of the other quantities. These laws can be combined to create a mathematical model for the physical situation. Once the model is found the challenge is to understand the "solution" to the model – often without actually having explicit formulas. The primary focus of this course is the mathematical analysis of differential equations. Students will learn a few special techniques to find analytic (but not necessarily explicit) solutions to differential equations.



Wuchen Li



Paula Vasquez Macias

MATH 527: Numerical Analysis (3 Credits)

Interpolation and approximation of functions; solution of algebraic equations; numerical differentiation and integration; numerical solutions of ordinary differential equations and boundary value problems; computer implementation of algorithms.

Prerequisites: C or better in MATH 520 or in both MATH 242 and MATH 344.

Cross-listed course: CSCE 561

Numerical Analysis studies the algorithms for the problems of continuous mathematics. The course will give an introduction to general ideas in Numerical Analysis and will discuss different aspects of the performance of the numerical procedures involved. In addition to the theoretical material, some numerical implementations in MATLAB will be considered on an elementary level. Topics include (not necessarily in the order they will be considered):

- number representations and loss of significance;
- polynomial interpolation;
- numerical differentiation;
- numerical integration;
- spline functions;
- method of least squares;
- numerical methods for ordinary differential equations;
- Monte Carlo methods.

At the end of this course students will be able to read, interpret, and use vocabulary, symbolism, and basic definitions from Numerical Analysis. The students will be able to use facts, formulas, and techniques learned in this course to apply algorithms and theorems to find numerical solutions and bounds on their errors to various types of problems including root finding, polynomial and spline approximation, numerical differentiation and integration, numerical solutions of ODEs.



Changhan He

MATH 528: Mathematical Foundation of Data Science and Machine Learning (3 Credits) Unconstrained and constrained optimization, gradient descent methods for numerical optimization, supervised and unsupervised learning, various reduced order methods, sampling and inference, Monte Carlo methods, deep neural networks.

Prerequisites: C or better in MATH 344 or MATH 544.

This course provides a mathematical introduction to principles underlying modern data science and machine learning methods. Students will develop an understanding of the core computational and statistical methods that power data-driven modeling and prediction. Emphasis is placed on the mathematical reasoning (linear algebra, calculus, optimization, probability, and statistics) that forms the foundation of common computational methods and algorithms. This will include linear and nonlinear regression, decision trees, ensemble learning, support vector machines, neural networks, and other widely used algorithms.

Students will develop a mathematical understanding of these algorithms, their implementation, and use in the evolving field of data science. Importantly, this class will provide both theoretical and practical insight into data science and machine learning through a combination of mathematical reasoning and computational examples primarily in Python. Through a combination of theoretical exploration and hands-on problem solving, students will gain the skills necessary to analyze, implement, and evaluate machine learning algorithms from first principles.



Mitchel Colebank

MATH 532: Modern Geometry (3 Credits)

Projective geometry, theorem of Desargues, conics, transformation theory, affine geometry, Euclidean geometry, non-Euclidean geometries, and topology.

Prerequisites: C or better in MATH 300.

The course focuses on two main topics: Non-Euclidean Geometry and modern approaches to Euclidean Geometry. The former topic will introduce the axioms of affine and projective planes and focus on establishing results from these axioms. As such, this part of the course will help build skills with proofs in mathematics. In addition, this part of the course will establish some concrete examples of affine and projective planes using modular arithmetic. The second part of the course will investigate the use of vectors, matrices, translations and rotations to establish interesting facts from Euclidean Geometry. The idea in this part of the course is to expose students to some fascinating material in Euclidean Geometry that goes beyond what one sees in a standard high school curriculum.



Daniel Savu

MATH 544: Linear Algebra (3 Credits)

Vectors, vector spaces, and subspaces; geometry of finite dimensional Euclidean space; linear transformations; eigenvalues and eigenvectors; diagonalization. Throughout there will be an emphasis on theoretical concepts, logic, and methods. MATH 544L is an optional laboratory course where additional applications will be discussed.

Prerequisites: C or better in MATH 241 and MATH 300.

Linear algebra is one of the fundamental topics in mathematics. Even if you do not know what linear algebra is, we have all been using many of the ideas for several years. While matrices will be common in this course, linear algebra is much more than "matrix algebra". A second and equally important objective of this course is the exposure to mathematical proofs. The early parts of the course emphasize manipulative aspects more than theoretical issues. As the course progresses, however, the same topics will be revisited — with more of an emphasis on the abstract theory of linear algebra. Students will master concepts and solve problems based on matrix algebra, solution of linear systems, notions of vector space, linear independence, basis, and dimension, linear transformations, change of basis, eigenvalues, eigenvectors, and diagonalization.

A solid knowledge of linear algebra – both manipulations and theory – will be helpful in almost any upper-division course in mathematics or any course that uses mathematics: differential equations, numerical analysis, optimization, etc.



Andrew Kustin



Wei-Lun Tsai

MATH 546: Algebraic Structures I (3 Credits)

Permutation groups; abstract groups; introduction to algebraic structures through study of subgroups, quotient groups, homomorphisms, isomorphisms, direct product; decompositions; introduction to rings and fields.

Prerequisites: C or better in MATH 300 and 544.

In this course, the student gets to experience mathematical thought beyond Calculus. As such, more sophistication is expected of the student. Most of the course will focus on group theory. Group theory is perhaps the area of mathematics with the fewest moving parts and the most ubiquity. Through studying group theory, each student will be exposed to the thought process involved in higher-level mathematics. Students will master concepts and solve problems based on permutation and abstract groups, subgroups, quotient groups, homomorphisms, isomorphisms, direct products, and rings.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.



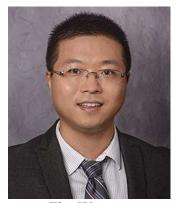
Adela Vraciu

MATH 550: Vector Analysis (3 Credits)

Vector fields, line and path integrals, orientation and parametrization of lines and surfaces, change of variables and Jacobians, oriented surface integrals, theorems of Green, Gauss, and Stokes; introduction to tensor analysis.

Prerequisites: C or better in MATH 241.

This is a continuation of Math 241 — Vector Calculus. The main objective is to understand, and apply, the three most important integral theorems of vector analysis: Green's, Stokes', and Gauss' Theorems. In preparation for these, there will be a brief review of paths, curves, vector fields, directional derivatives, gradients, divergence, and curl. Next, we will cover maps, change of variables, multiple integration, and parameterized surfaces as well as line, path, and surface integrals. By the end of the semester, students will be able to exploit algebraic and geometric methods to compute integrals using the three big theorems.



Zhu Wang



Lili Ju

MATH 552: Applied Complex Variables (3 Credits)

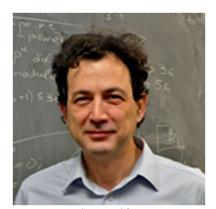
Complex integration, calculus of residues, conformal mapping, Taylor and Laurent Series expansions, applications.

Prerequisites: C or better in MATH 241.

The emphasis of this course will be on the analysis of functions whose domain and/or range are sets of complex numbers. Much of this analysis will be very similar to the real-valued calculus that is the prerequisite for this course. Another objective is to define versions of elementary functions when the argument is a complex number. The "new" functions should be consistent with their real-valued counterparts and should maintain all of the usual properties.

The Cauchy Integral Theorem is one of the major triumphs of complex analysis. This theorem can be viewed as an extension of Green's Theorem (which provided a connection between double integrals and line integrals). One of the most important applications of the Cauchy Integral Theorem is the easy evaluation of many contour integrals.

Students will master the fundamental concepts from Complex Analysis, including the concept of a holomorphic function, complex line integrals, Cauchy's Theorem, Cauchy's Integral Formula, classification of zeros and singularities, and applications to residue calculus.



Ognian Trifonov

MATH 554: Analysis I (3 Credits)

Least upper bound axiom, the real numbers, compactness, sequences, continuity, uniform continuity, differentiation, Riemann integral and fundamental theorem of calculus.

Prerequisites: C or better in MATH 241 and two 500-level classes requiring MATH 300: MATH 525, MATH 531, MATH 532, MATH 533, MATH 534, MATH 540, MATH 541, MATH 544, MATH 546, MATH 548, MATH 551, MATH 561, MATH 570, MATH 574, MATH 575, or MATH 580.

In this course, you will learn the proofs of concepts you used in the computations done in Calculus I and II. Thus, in this course you will be writing many proofs (thus the prerequisite).

While most of science is based on inductive reasoning, mathematics is based on deductive reasoning. This means that new results are formed from logical combinations of hypothesis and statements accepted as true. Every result and technique learned in calculus (and other mathematics courses) is logically consistent and can be derived in a rigorous manner. In this course students begin to study some basic properties used to develop the fundamental calculus results including convergence of sequences, limit of a function, continuity (point-wise and uniform), derivative of a function, Rolle's theorem and the mean value theorem, L'Hospital's rule, inverse function theorem, Riemann integrals, Fundamental Theorem of Calculus, and derivatives of integrals. To be able to understand these results, and their proofs, it is necessary to develop a solid foundation in the real number system. It is also necessary to develop the ability to read, understand and write mathematical proofs. One of the most important steps in the creation of a mathematical proof is a solid understanding of the basic definitions. Unlike most previous courses you have taken, it is essential to pay attention to the details and technicalities. While this may be slightly unnatural, it is a skill that can be acquired through practice and patience.

Students will become knowledgeable about and will master concepts of real analysis. Students will improve their ability to write and read mathematical proofs, particularly those related to the least upper bound axiom, compactness, sequences, continuity, uniform continuity, differentiation, Riemann integration, and the Fundamental Theorem of Calculus.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.



Maria Girardi

MATH 555: Analysis II (3 Credits)

Riemann-Stieltjes integral, infinite series, sequences and series of functions, uniform convergence, Weierstrass approximation theorem, selected topics from Fourier series or Lebesgue integration.

Prerequisites: C or better in MATH 554



Kyle Liss

MATH 574: Discrete Mathematics I (3 Credits)

Mathematical models; mathematical reasoning; enumeration; induction and recursion; tree structures; networks and graphs; analysis of algorithms.

Prerequisites: C or better in MATH 300.

Students will master concepts and solve problems in discrete mathematics, including basic set theory, counting, relations, and graphs. The use of the proof techniques learned earlier will be reinforced throughout the class. Students will master the concepts and be able to solve problems associated with enumeration, permutations and combinations, recurrence relations, and the groundwork for the more advanced topics of graph theory and game theory.



Ruth Luo

MATH 576: Combinatorial Game Theory (3 Credits)

Winning in certain combinatorial games such as Nim, Hackenbush, and Domineering. Equalities and inequalities among games, Sprague-Grundy theory of impartial games, games which are numbers.

Prerequisites: C or better in MATH 300 or MATH 374.

Through this course, students learn the winning strategy in certain combinatorial games such as Nim, Hackenbush, and Domineering. Students will learn equalities and inequalities among games, Sprague-Gundy theory of impartial games, and games which are numbers.



William Linz