Pi Mu Epsilon

& GAMECOCK MATH CLUB

Getting to Know Your 500-Level MATH Courses at the University of South Carolina

Event sponsored in part by

UNIVERSITY OF SOUTH CAROLINA

Department of Mathematics

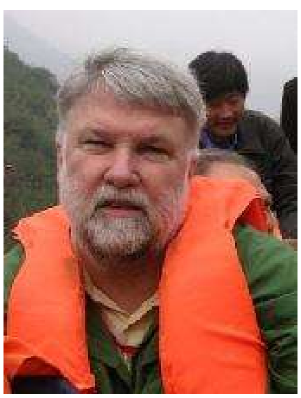
MATH 511: Probability (3 Credits)
Probability and independence; discrete and continuous random variables; joint, marginal, and conditional densities, moment generating functions; laws of large numbers; binomial, Poisson, gamma, univariate, and bivariate normal distributions.

Prerequisite or Corequisite: C or better in MATH 241.
Cross-listed course: STAT 511

The purpose of this course is to give you an introduction to probablity theory and probablity distributions. The material presented will not only serve as a basis for the subsequent courses, STAT 512/513, but is also extremely useful and fascinating in its own right. STAT 511 has a prerequisite of a standard multivariable calculus course, and a strong familiarity with differentitation, integration, infinite series and sequences, and related facts, is necessary. This course is very important for those of you considering careers in actuarial sciences.

The course covers the axiomatic approach to probability, counting techniques, Bayes Theorem, random variables, probability distributions for discrete and continuous random variables, mathematical expectation, moment generating functions, joint and conditional distributions for multiple random variables, and measures of association (covariance and correlation). This course focuses on both theory and application. You will be expected to derive theoretical results using algebra and calculus and apply these results to problems from a multitude of applications.

Spring 2025 Professor:



Ralph Howard

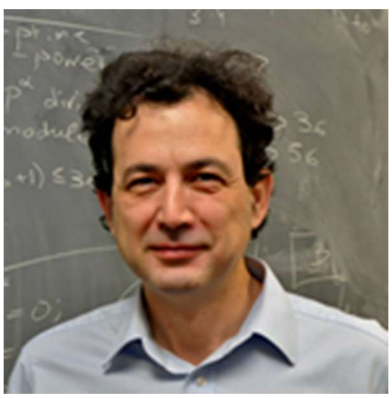
MATH 520: Ordinary Differential Equations (3 Credits)
Differential equations of the first order, linear systems of ordinary differential equations, elementary qualitative properties of nonlinear systems.

Prerequisites: C or better in MATH 344 or MATH 544.

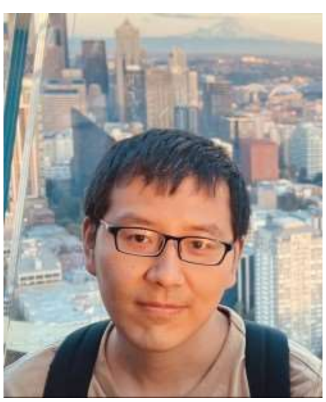
Upon completion of this course, students will be knowledgeable about and will be able to analyze solutions to differential equations of the first order and linear systems of ordinary differential equations. They will also be able to apply these ideas to determine elementary qualitative properties of nonlinear systems.

Differential equations is the language of science. Many basic scientific laws express the change in one quantity in terms of the values of the other quantities. These laws can be combined to create a mathematical model for the physical situation. Once the model is found the challenge is to understand the "solution" to the model – often without actually having explicit formulas. The primary focus of this course is the mathematical analysis of differential equations. Students will learn a few special techniques to find analytic (but not necessarily explicit) solutions to differential equations.

Spring 2025 Professors:



Ognian Trifonov



Haonan Zhang

MATH 521: Boundary Value Problems and Partial Differential Equations (3 Credits)
Laplace transforms, two-point boundary value problems and Green's functions, boundary value problems in partial differential equations, eigenfunction expansions and separation of variables, transform methods for solving PDE's, Green's functions for PDE's, and the method of characteristics.

Prerequisites: C or better in MATH 520 or in both MATH 241 and MATH 242.

Many principles and laws underlying the behavior of the natural world are given by differential equations. To understand and to investigate the problems in the natural world, it is imperative to solve the differential equations or to understand their properties. This course aims at providing students with elementary methods of solving simple partial differential equations (PDE). The scope of the course covers the methods for first order linear partial differential equations, heat, wave, and Poisson equations and some applications involving these partial differential equations. Boundary value problems and Fourier methods are introduced to develop efficient methods for solving partial differential equations. Students will master basic concepts and methods for solving first order linear, heat, wave, and Poisson equations and associated eigenvalue problems.

Spring 2025 Professor:

Siming He

MATH 523: **Mathematical Modeling of Population Biology (3 Credits)**
Applications of differential and difference equations and linear algebra modeling the dynamics of populations, with emphasis on stability and oscillation. Critical analysis of current publications with computer simulation of models.

Prerequisites: C or better in MATH 142, BIOL 301, or MSCI 311 recommended.

Spring 2025 Professor:

Paula Vasquez

MATH 527: Numerical Analysis (3 Credits)
Interpolation and approximation of functions; solution of algebraic equations; numerical differentiation and integration; numerical solutions of ordinary differential equations and boundary value problems; computer implementation of algorithms.

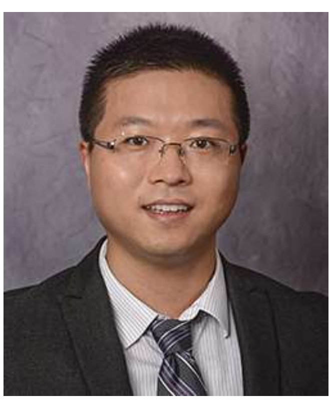
Prerequisites: C or better in MATH 520 or in both MATH 242 and MATH 344.
Cross-listed course: CSCE 561

Numerical Analysis studies the algorithms for the problems of continuous mathematics. The course will give an introduction to general ideas in Numerical Analysis and will discuss different aspects of the performance of the numerical procedures involved. In addition to the theoretical material, some numerical implementations in MATLAB will be considered on an elementary level. Topics include (not necessarily in the order they will be considered):

- number representations and loss of significance;
- polynomial interpolation;
- numerical differentiation;
- numerical integration;
- spline functions;
- method of least squares;
- numerical methods for ordinary differential equations;
- Monte Carlo methods.

At the end of this course students will be able to read, interpret, and use vocabulary, symbolism, and basic definitions from Numerical Analysis. The students will be able to use facts, formulas, and techniques learned in this course to apply algorithms and theorems to find numerical solutions and bounds on their errors to various types of problems including root finding, polynomial and spline approximation, numerical differentiation and integration, numerical solutions of ODEs.

Spring 2025 Professor:

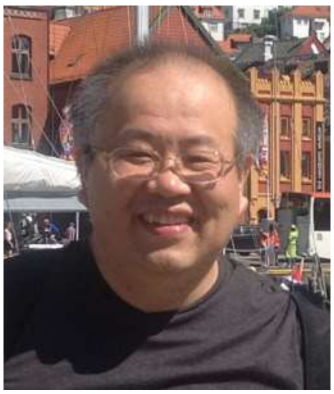


Zhu Wang

MATH 528: **Mathematical Foundation of Data Science and Machine Learning (3 Credits)**
Unconstrained and constrained optimization, gradient descent methods for numerical optimization, supervised and unsupervised learning, various reduced order methods, sampling and inference, Monte Carlo methods, deep neural networks.

Prerequisites: C or better in MATH 344 or MATH 544.

Spring 2025:



Hong Wang

MATH 532 - Modern Geometry (3 Credits)
Projective geometry, theorem of Desargues, conics, transformation theory, affine geometry, Euclidean geometry, non-Euclidean geometries, and topology.

Prerequisites: C or better in MATH 300.

The course focuses on two main topics: Non-Euclidean Geometry and modern approaches to Euclidean Geometry. The former topic will introduce the axioms of affine and projective planes and focus on establishing results from these axioms. As such, this part of the course will help build skills with proofs in mathematics. In addition, this part of the course will establish some concrete examples of affine and projective planes using modular arithmetic. The second part of the course will investigate the use of vectors, matrices, translations and rotations to establish interesting facts from Euclidean Geometry. The idea in this part of the course is to expose students to some fascinating material in Euclidean Geometry that goes beyond what one sees in a standard high school curriculum.

Spring 2025 Professor:

Daniel Savu

MATH 544: Linear Algebra (3 Credits)
Vectors, vector spaces, and subspaces; geometry of finite dimensional Euclidean space; linear transformations; eigenvalues and eigenvectors; diagonalization. Throughout there will be an emphasis on theoretical concepts, logic, and methods. MATH 544L is an optional laboratory course where additional applications will be discussed.

Prerequisites: C or better in MATH 241 and MATH 300.

Linear algebra is one of the fundamental topics in mathematics. Even if you do not know what linear algebra is, we have all been using many of the ideas for several years. While matrices will be common in this course, linear algebra is much more than "matrix algebra". A second and equally important objective of this course is the exposure to mathematical proofs. The early parts of the course emphasize manipulative aspects more than theoretical issues. As the course progresses, however, the same topics will be revisited – with more of an emphasis on the abstract theory of linear algebra. Students will master concepts and solve problems based on matrix algebra, solution of linear systems, notions of vector space, linear independence, basis, and dimension, linear transformations, change of basis, eigenvalues, eigenvectors, and diagonalization.

A solid knowledge of linear algebra – both manipulations and theory – will be helpful in almost any upper-division course in mathematics or any course that uses mathematics: differential equations, numerical analysis, optimization, etc.

Spring 2025 Professors:

Geoffrey Dillon

Frank Thorne

MATH 546: Algebraic Structures I (3 Credits)
Permutation groups; abstract groups; introduction to algebraic structures through study of subgroups, quotient groups, homomorphisms, isomorphisms, direct product; decompositions; introduction to rings and fields.

Prerequisites: C or better in MATH 300 and 544.

In this course, the student gets to experience mathematical thought beyond Calculus. As such, more sophistication is expected of the student. Most of the course will focus on group theory. Group theory is perhaps the area of mathematics with the fewest moving parts and the most ubiquity. Through studying group theory, each student will be exposed to the thought process involved in higher-level mathematics. Students will master concepts and solve problems based on permutation and abstract groups, subgroups, quotient groups, homomorphisms, isomorphisms, direct products, and rings.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.

Spring 2025 Professor:

Matthew Boylan

MATH 547: Algebraic Structures II (3 Credits)
Rings, ideals, polynomial rings, unique factorization domains; structure of finite groups; topics from: fields, field extensions, Euclidean constructions, modules over principal ideal domains (canonical forms).

Prerequisites: C or better in MATH 546.

MATH 547 is the continuation of MATH 546. MATH 546 is about groups, while MATH 547 is about rings and fields. A field is a set F with two operations, usually called addition and multiplication. Under addition, F is an abelian group, with an identity element called 0. Under multiplication, $F\setminus\{0\}$, is an abelian group. The distributive axiom describes the interplay between the two operations. A ring is a set with two operations. Some of the field axioms hold in a ring. Some examples of fields are: the set of rational numbers, the set of real numbers, and the set of complex numbers. Every field is automatically a ring. The set of integers is a good example of a ring which is not a field. If R is a ring, then the set of all polynomials $\{f(x)\}$ with coefficients from R is another ring.

Students in this course will master concepts and solve problems on rings: ideals, polynomial rings, Euclidean domains, unique factorization domains; fields: extensions, Galois Theory, Euclidean constructions: modules over principal ideal domains.

Spring 2025 Professor:

Adela Vraciu

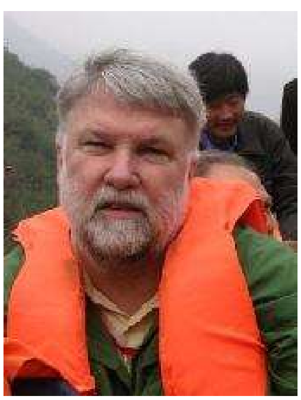
MATH 550: Vector Analysis (3 Credits)
Vector fields, line and path integrals, orientation and parametrization of lines and surfaces, change of variables and Jacobians, oriented surface integrals, theorems of Green, Gauss, and Stokes; introduction to tensor analysis.

Prerequisites: C or better in MATH 241.

This is a continuation of Math 241 — Vector Calculus. The main objective is to understand, and apply, the three most important integral theorems of vector analysis: Green's, Stokes', and Gauss' Theorems. In preparation for these, there will be a brief review of paths, curves, vector fields, directional derivatives, gradients, divergence, and curl. Next, we will cover maps, change of variables, multiple integration, and parameterized surfaces as well as line, path, and surface integrals. By the end of the semester, students will be able to exploit algebraic and geometric methods to compute integrals using the three big theorems.

Spring 2025 Professors:

Xinfeng Liu



Ralph Howard

MATH 552: Applied Complex Variables (3 Credits)
Complex integration, calculus of residues, conformal mapping, Taylor and Laurent Series expansions, applications.

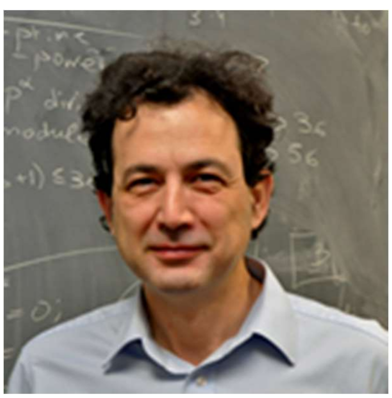
Prerequisites: C or better in MATH 241.

The emphasis of this course will be on the analysis of functions whose domain and/or range are sets of complex numbers. Much of this analysis will be very similar to the real-valued calculus that is the prerequisite for this course. Another objective is to define versions of elementary functions when the argument is a complex number. The “new” functions should be consistent with their real-valued counterparts and should maintain all of the usual properties.

The Cauchy Integral Theorem is one of the major triumphs of complex analysis. This theorem can be viewed as an extension of Green’s Theorem (which provided a connection between double integrals and line integrals). One of the most important applications of the Cauchy Integral Theorem is the easy evaluation of many contour integrals.

Students will master the fundamental concepts from Complex Analysis, including the concept of a holomorphic function, complex line integrals, Cauchy’s Theorem, Cauchy’s Integral Formula, classification of zeros and singularities, and applications to residue calculus.

Spring 2025 Professor:



Ognian Trifonov

MATH 554: Analysis I (3 Credits)
Least upper bound axiom, the real numbers, compactness, sequences, continuity, uniform continuity, differentiation, Riemann integral and fundamental theorem of calculus.

Prerequisites: C or better in MATH 241 and two 500-level classes requiring MATH 300: MATH 525, MATH 531, MATH 532, MATH 533, MATH 534, MATH 540, MATH 541, MATH 544, MATH 546, MATH 548, MATH 551, MATH 561, MATH 570, MATH 574, MATH 575, or MATH 580.

In this course, you will learn the proofs of concepts you used in the computations done in Calculus I and II. Thus, in this course you will be writing many proofs (thus the prerequisite).

While most of science is based on inductive reasoning, mathematics is based on deductive reasoning. This means that new results are formed from logical combinations of hypothesis and statements accepted as true. Every result and technique learned in calculus (and other mathematics courses) is logically consistent and can be derived in a rigorous manner. In this course students begin to study some basic properties used to develop the fundamental calculus results including convergence of sequences, limit of a function, continuity (point-wise and uniform), derivative of a function, Rolle's theorem and the mean value theorem, L'Hospital's rule, inverse function theorem, Riemann integrals, Fundamental Theorem of Calculus, and derivatives of integrals. To be able to understand these results, and their proofs, it is necessary to develop a solid foundation in the real number system. It is also necessary to develop the ability to read, understand and write mathematical proofs. One of the most important steps in the creation of a mathematical proof is a solid understanding of the basic definitions. Unlike most previous courses you have taken, it is essential to pay attention to the details and technicalities. While this may be slightly unnatural, it is a skill that can be acquired through practice and patience.

Students will become knowledgeable about and will master concepts of real analysis. Students will improve their ability to write and read mathematical proofs, particularly those related to the least upper bound axiom, compactness, sequences, continuity, uniform continuity, differentiation, Riemann integration, and the Fundamental Theorem of Calculus.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.

Spring 2025 Professor:

Maria Girardi

MATH 574: Discrete Mathematics I (3 Credits)
Mathematical models; mathematical reasoning; enumeration; induction and recursion; tree structures; networks and graphs; analysis of algorithms.

Prerequisites: C or better in MATH 300.

Students will master concepts and solve problems in discrete mathematics, including basic set theory, counting, relations, and graphs. The use of the proof techniques learned earlier will be reinforced throughout the class. Students will master the concepts and be able to solve problems associated with enumeration, permutations and combinations, recurrence relations, and the groundwork for the more advanced topics of graph theory and game theory.

Spring 2025 Professor:

Eva Czabarka

MATH 575: Discrete Mathematics II (3 Credits)
A continuation of MATH 574. Inversion formulas; Polya counting; combinatorial designs; minimax theorems; probabilistic methods; Ramsey theory; other topics.

Prerequisites: C or better in MATH 574.

Students will make progress with logical thinking, communicating mathematical ideas, and developing problem-solving skills, by

- writing up solutions to a wide range of homework exercises,
- learning and writing up proofs to sophisticated classical theorems on tests, and
- presenting solutions or projects to class.

We will prove several of the central discoveries of Graph Theory, learn statements of others that are too advanced to prove in this course, and think about complexity of algorithms for related problems. Among the topics we would like to cover this semester are:

1. Basic terminology and concepts for simple undirected graphs
2. Isomorphism, connectivity, degree sequences, spanning trees, cliques, independent sets
3. Euler and Hamilton paths and cycles
4. Extremal graph theory: Mantel's Theorem, Turan's Theorem
5. Matching theory: Hall's Theorem, Tutte's Theorem
6. Graph coloring: Brooks's Theorem
7. Lambda labellings: Delta-squared Conjecture
8. Planar graphs: Euler's formula, Kuratowski's Theorem, 5-Color Theorem, 4-Color Theorem
9. Trees, spanning tree

As time allows, students will be introduced to Directed graphs, multigraphs, hypergraphs, Ramsey Theory, Connectivity (Menger's Theorem), and Network flows

Spring 2025 Professor:

William Linz

MATH 576: Combinatorial Game Theory (3 Credits)
Winning in certain combinatorial games such as Nim, Hackenbush, and Domineering. Equalities and inequalities among games, Sprague-Grundy theory of impartial games, games which are numbers.

Prerequisites: C or better in MATH 300 or MATH 374.

Through this course, students learn the winning strategy in certain combinatorial games such as Nim, Hackenbush, and Domineering. Students will learn equalities and inequalities among games, Sprague-Gundy theory of impartial games, and games which are numbers.

Spring 2025 Professor:

William Linz