Pi Mu Epsilon
& GAMECOCK MATH CLUB

Getting to Know Your 500-Level MATH Courses at the University of South Carolina

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MATH 511/STAT 511: Probability

Prerequisite: Grade of C or higher or concurrent enrollment in MATH 241

The purpose of this course is to give you an introduction to probablity theory and probablity distributions. The material presented will not only serve as a basis for the subsequent courses, STAT 512/513, but is also extremely useful and fascinating in its own right. STAT 511 has a prerequisite of a standard multivariable calculus course, and a strong familiarity with differentitation, integration, infinite series and sequences, and related facts, is necessary. This course is very important for those of you considering careers in actuarial sciences.

The course covers the axiomatic approach to probability, counting techniques, Bayes Theorem, random variables, probability distributions for discrete and continuous random variables, mathematical expectation, moment generating functions, joint and conditional distributions for multiple random variables, and measures of association (covariance and correlation). This course focuses on both theory and application. You will be expected to derive theoretical results using algebra and calculus and apply these results to problems from a multitude of applications.

Spring 2018 Professor:

Jesse Kass

MATH 515 (STAT 523): Financial Mathematics II

Prerequisite: Grade of C or better in MATH 514 (STAT 522)

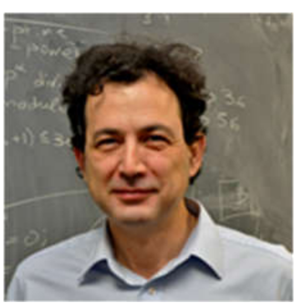
Complex mathematical techniques are now widely used in finance. This course is the second part of a two course sequence that provides an elementary introduction to these techniques and to the fundamental concepts of financial mathematics. We examine some mathematical models that are used in finance, e.g. to model risk and return of financial assets and to model the random nature of stock prices.

In the first part of the course we continue our investigation of the `No Arbitrage Principle'. A very important mathematical result known as the Separating Hyperplane Theorem will be proved in order to establish the `Fundamental Theorem of Asset Pricing' and the closely related `Arbitrage Theorem'. We also consider some other interesting mathematical consequences of the Separating Hyperplane Theorem such as the existence of a steady state vector for Markov matrices.

In the second part of the course we look at some mathematical problems arising in portfolio selection. Taking into account the investor's tolerance for risk leads to the problem of maximizing expected `utility'. Using the technique of `mean-variance analysis' we derive the main results of the famous Capital Asset Pricing Model which relates the riskiness of an individual stock to the overall market through a numerical coefficient known as `beta'. If time permits we will consider some discrete optimization problems such as the `knapsack problem' which arise in portfolio selection.

In the third part of the course we shall return to the problem of options pricing. Using the multi-period binomial model we shall find an algorithm for the pricing of the American put option. The existence of an `equivalent martingale measure' for which the discounted stock process is a `fair game' leads us naturally to the general theory of martingales. We shall examine some of the basic concepts and results of this theory such as the notion of a `stopping time' and Doob's Optional Sampling Theorem.

Spring 2018 Professor:



Ognian Trifonov

MATH 520: Ordinary Differential Equations

Prerequisite: Grade of C or better in MATH 344 or MATH 544

Upon completion of this course, students will be knowledgeable about and will be able to analyze solutions to differential equations of the first order and linear systems of ordinary differential equations. They will also be able to apply these ideas to determine elementary qualitative properties of nonlinear systems.

Differential equations is the language of science. Many basic scientific laws express the change in one quantity in terms of the values of the other quantities. These laws can be combined to create a mathematical model for the physical situation. Once the model is found the challenge is to understand the "solution" to the model – often without actually having explicit formulas. The primary focus of this course is the mathematical analysis of differential equations. Students will learn a few special techniques to find analytic (but not necessarily explicit) solutions to differential equations.

Spring 2018 Professors:

Xinfeng Liu

Paula Vasquez

MATH 521: Boundary Value Problems and Partial Differential Equations

Prerequisite: MATH 520 or MATH 241 and 242

Many principles and laws underlying the behavior of the natural world are given by differential equations. To understand and to investigate the problems in the natural world, it is imperative to solve the differential equations or to understand their properties. This course aims at providing students with elementary methods of solving simple partial differential equations (PDE). The scope of the course covers the methods for first order linear partial differential equations, heat, wave, and Poisson equations and some applications involving these partial differential equations. Boundary value problems and Fourier methods are introduced to develop efficient methods for solving partial differential equations. Students will master basic concepts and methods for solving first order linear, heat, wave, and Poisson equations and associated eigenvalue problems.

Spring 2018 Professor:

Doug Meade

MATH 522: Wavelets

Prerequisite: Grade of C or higher in MATH 544

The course will give an introduction to Fourier transform, wavelets and multiresolution analysis. While the basic theory will be briefly introduced, the emphasis will be given to the description of the general ideas and the numerical procedures. In addition to the theoretical material some numerical implementations in MATLAB will be considered on an elementary level. Topics include (not necessarily in the order they will be considered):
- signals and filters;
- lifting scheme;
- Haar wavelets;
- Daubechies wavelets;
- inner product spaces and orthogonalization;
- Fourier transform and Fast Fourier Transform;
- multiresolution analysis;
- numerical implementation of the wavelet transform.

At the end of this course students will master concepts of multiresolution analysis, digital filters, and wavelets. They will be able to interpret the outcomes of digital wavelet transform (DWT) and to design basic numerical algorithms for DWT in MATLAB.

Spring 2018 Professor:

Pencho Petrushev

MATH 527 (CSCE 561): Numerical Analysis

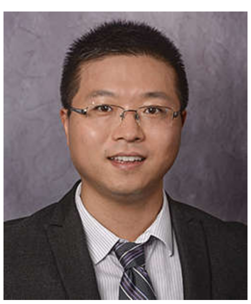
Prerequisite: Grade of C or better in MATH 520 or in both MATH 242 and MATH 344

Numerical Analysis studies the algorithms for the problems of continuous mathematics. The course will give an introduction to general ideas in Numerical Analysis and will discuss different aspects of the performance of the numerical procedures involved. In addition to the theoretical material, some numerical implementations in MATLAB will be considered on an elementary level. Topics include (not necessarily in the order they will be considered):

- number representations and loss of significance;
- polynomial interpolation;
- numerical differentiation;
- numerical integration;
- spline functions;
- method of least squares;
- numerical methods for ordinary differential equations;
- Monte Carlo methods.

At the end of this course students will be able to read, interpret, and use vocabulary, symbolism, and basic definitions from Numerical Analysis. The students will be able to use facts, formulas, and techniques learned in this course to apply algorithms and theorems to find numerical solutions and bounds on their errors to various types of problems including root finding, polynomial and spline approximation, numerical differentiation and integration, numerical solutions of ODEs.

Spring 2018 Professor:



Zhu Wang

MATH 532: Modern Geometry

Prerequisite: Grade of C or higher in MATH 300

The course focuses on two main topics: Non-Euclidean Geometry and modern approaches to Euclidean Geometry. The former topic will introduce the axioms of affine and projective planes and focus on establishing results from these axioms. As such, this part of the course will help build skills with proofs in mathematics. In addition, this part of the course will establish some concrete examples of affine and projective planes using modular arithmetic. The second part of the course will investigate the use of vectors, matrices, translations and rotations to establish interesting facts from Euclidean Geometry. The idea in this part of the course is to expose students to some fascinating material in Euclidean Geometry that goes beyond what one sees in a standard high school curriculum.

Spring 2018 Professor:

Xian Wu

MATH 544: Linear Algebra

Prerequisite: Grade of C or higher in MATH 300

Linear algebra is one of the fundamental topics in mathematics. Even if you do not know what linear algebra is, we have all been using many of the ideas for several years. While matrices will be common in this course, linear algebra is much more than "matrix algebra". A second and equally important objective of this course is the exposure to mathematical proofs. The early parts of the course emphasize manipulative aspects more than theoretical issues. As the course progresses, however, the same topics will be revisited – with more of an emphasis on the abstract theory of linear algebra. Students will master concepts and solve problems based on matrix algebra, solution of linear systems, notions of vector space, linear independence, basis, and dimension, linear transformations, change of basis, eigenvalues, eigenvectors, and diagonalization.

A solid knowledge of linear algebra – both manipulations and theory – will be helpful in almost any upper-division course in mathematics or any course that uses mathematics: differential equations, numerical analysis, optimization, etc.

Spring 2018 Professors:

Patrick McFaddin

George McNulty

MATH 546: Algebraic Structures I

Prerequisite: Grade of C or higher in MATH 544

In this course, the student gets to experience mathematical thought beyond Calculus. As such, more sophistication is expected of the student. Most of the course will focus on group theory. Group theory is perhaps the area of mathematics with the fewest moving parts and the most ubiquity. Through studying group theory, each student will be exposed to the thought process involved in higher-level mathematics. Students will master concepts and solve problems based on permutation and abstract groups, subgroups, quotient groups, homomorphisms, isomorphisms, direct products, and rings.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.

Spring 2018 Professor:

Adela Vraciu

MATH 547H: Algebraic Structures II

Prerequisite: Grade of C or better in MATH 546

MATH 547H is the continuation of MATH 546H. We will start by studying rings; these are structures exemplified by the integers or by polynomials with rational or real coefficients, which carry two interacting operations: addition and multiplication. We will study different types of substructures (subrings, ideals), and concepts generalizing the idea of prime numbers and factorization. Just as with groups the operation preserving maps between objects reflect structural properties of the objects. Fields are a special type of ring in which all non-zero elements are invertible; we'll mimic the construction of the rationals from the integers by constructing the quotient field of any integral domain, and we'll mimic the construction of the complex numbers from the reals by constructing algebraic extensions in general. Group theory and field theory come together in Galois theory, which concerns the ancient question of the possibility of expressing roots of polynomials in terms of radicals.

Spring 2018 Professor:

Matt Miller

MATH 548: Geometry, Algebra, and Algorithms

Prerequisites: MATH 300 and MATH 544 or consent of the Undergraduate Director.

The course is an introduction to algebraic geometry - the study of systems of polynomial equations - with a hands-on computational emphasis. Students will learn about Gröbner bases and Buchberger's algorithm, which are a nonlinear generalization of Gaussian elimination from linear algebra. A key application is a practical method for solving polynomial equations; indeed, versions of the algorithms studied in the course are used in most computer algebra software.

For those considering graduate school in mathematics, this course is an introduction to algebraic geometry, commutative algebra, and symbolic algebra. Additionally, the course is excellent training for mathematics REU opportunities as computer algebra is a common feature of many of these programs. Students with more applied interests will be well-positioned to learn applications to image processing, robotics, genetics, and the new field of algebraic statistics.

The official prerequisites are Math 300 and Math 544 since some comfort with proof and linear algebra is assumed. However, for strong students, Math 374 may be an adequate substitute for Math 300, while Math 344 or 526 may be a replacement for Math 544. Computer algebra systems will be used during the course, but no programming experience is required.

Spring 2018 Professor:

Alex Duncan

MATH 550: Vector Analysis

Prerequisite: Grade of C or higher in MATH 241

This course is an extension of MATH 241 and has much the same problem solving character with a dose of theory and abstraction. Proofs will be given in class and you will be expected to do some computationally driven proofs on your own, but the course will not have the same proof oriented character as a course like MATH 554. This course makes a nice transition from calculus to higher-level theoretical math courses.

After a quick review of MATH 241, the main material of the course consists of the "Big Three" Theorems of Green, Stokes, and Gauss and how to use them. Students will demonstrate an understanding of vector functions by solving problems in the context of vector fields (e.g., by distinguishing gradient fields from non-gradient fields), line integrals, surface integrals, divergence and curl. Students will be able to exploit algebraic and geometric methods to compute integrals using the theorems of Green, Stokes, and Gauss, as well as direct computation using parameterizations.

Spring 2018 Professors:

Peter Binev

Doug Meade

MATH 552: Applied Complex Analysis

Prerequisite: Grade of C or higher in MATH 241

The emphasis of this course will be on the analysis of functions whose domain and/or range are sets of complex numbers. Much of this analysis will be very similar to the real-valued calculus that is the prerequisite for this course. Another objective is to define versions of elementary functions when the argument is a complex number. The "new" functions should be consistent with their real-valued counterparts and should maintain all of the usual properties.

The Cauchy Integral Theorem is one of the major triumphs of complex analysis. This theorem can be viewed as an extension of Green's Theorem (which provided a connection between double integrals and line integrals). One of the most important applications of the Cauchy Integral Theorem is the easy evaluation of many contour integrals.

Students will master the fundamental concepts from Complex Analysis, including the concept of a holomorphic function, complex line integrals, Cauchy's Theorem, Cauchy's Integral Formula, classification of zeros and singularities, and applications to residue calculus.

Spring 2018 Professor:

Daniel Dix

MATH 554: Analysis I

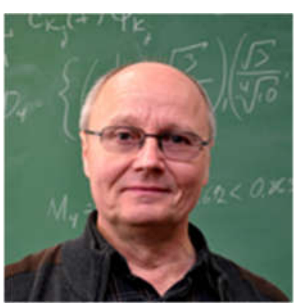
Prerequisite: Grade of C or higher in MATH 300 and at least one of MATH 511, 520, 534, 550, or 552

While most of science is based on inductive reasoning, mathematics is based on deductive reasoning. This means that new results are formed from logical combinations of hypothesis and statements accepted as true. Every result and technique learned in calculus (and other mathematics courses) is logically consistent and can be derived in a rigorous manner. In this course students begin to study some basic properties used to develop the fundamental calculus results including convergence of sequences, limit of a function, continuity (point-wise and uniform), derivative of a function, Rolle's theorem and the mean value theorem, L'Hospital's rule, inverse function theorem, Riemann integrals, Fundamental Theorem of Calculus, and derivatives of integrals. To be able to understand these results, and their proofs, it is necessary to develop a solid foundation in the real number system. It is also necessary to develop the ability to read, understand and write mathematical proofs. One of the most important steps in the creation of a mathematical proof is a solid understanding of the basic definitions. Unlike most previous courses you have taken, it is essential to pay attention to the details and technicalities. While this may be slightly unnatural, it is a skill that can be acquired through practice and patience.

Students will become knowledgeable about and will master concepts of real analysis. Students will improve their ability to write and read mathematical proofs, particularly those related to the least upper bound axiom, compactness, sequences, continuity, uniform continuity, differentiation, Riemann integration, and the Fundamental Theorem of Calculus.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.

Spring 2018 Professor:



Vladimir Temlyakov

MATH 555: Analysis II

Prerequisite: Grade of C or better in MATH 554

This course is a continuation of Math 554, Analysis I. If you enjoyed Math 554, you should strongly consider taking this course. It is also a must for anyone who is interested in pursuing graduate work in fields which use Mathematics. Topics which we will cover in this course include uniform continuity, derivatives, the Mean Value Theorem, L'Hopital's Rule, Riemann integrals, Riemann integrability, the Fundamental Theorem of Caluclus, integration by parts, sequences and series, pointwise convergence, uniform convergence, metric spaces, sets of measure zero, and a connection between the latter and Riemann integrability.

Spring 2018 Professor:

Anton Schep

MATH 562 (CSCE 551): Theory of Computation

Prerequisite: C or better in CSCE 350 or MATH 300, 344 or 544 or 574, or consent of the Undergraduate Director

This course teaches the basic theoretical principles of computing. Computing tasks and devices are modeled as mathematical objects (such as formal languages, automata, and Turing machines) so that we can prove facts about their abilities and limitations. We show that some problems are not solvable by computer, regardless of how much time and memory are used. We also address issues of efficient computation and show that many interesting problems such as the Traveling Salesman problem, though solvable, are unlikely to be solved efficiently. Topics include: finite automata, formal languages, Turing machines, decidability and undecidability, reductions between problems, complexity classes P, NP, and PSPACE, NP-completeness, and PSPACE-completeness.

Spring 2018 Professor:

Stephen Fenner

MATH 574: Discrete Mathematics

Prerequisite: Grade of C or higher in MATH 300

Students will master concepts and solve problems in discrete mathematics, including basic set theory, counting, relations, and graphs. The use of the proof techniques learned earlier will be reinforced throughout the class. Students will master the concepts and be able to solve problems associated with enumeration, permutations and combinations, recurrence relations, and the groundwork for the more advanced topics of graph theory and game theory.

Spring 2018 Professor:

Eva Czabarka

MATH 575: Discrete Mathematics II

Prerequisite: Grade of C or better in MATH 574

Students will make progress with logical thinking, communicating mathematical ideas, and developing problem-solving skills, by

- writing up solutions to a wide range of homework exercises,
- learning and writing up proofs to sophisticated classical theorems on tests, and
- presenting solutions or projects to class.

We will prove several of the central discoveries of Graph Theory, learn statements of others that are too advanced to prove in this course, and think about complexity of algorithms for related problems. Among the topics we would like to cover this semester are:

1. Basic terminology and concepts for simple undirected graphs
2. Isomorphism, connectivity, degree sequences, spanning trees, cliques, independent sets
3. Euler and Hamilton paths and cycles
4. Extremal graph theory: Mantel's Theorem, Turan's Theorem
5. Matching theory: Hall's Theorem, Tutte's Theorem
6. Graph coloring: Brooks's Theorem
7. Lambda labellings: Delta-squared Conjecture
8. Planar graphs: Euler's formula, Kuratowski's Theorem, 5-Color Theorem, 4-Color Theorem
9. Trees, spanning tree

As time allows, students will be introduced to Directed graphs, multigraphs, hypergraphs, Ramsey Theory, Connectivity (Menger's Theorem), and Network flows

Spring 2018 Professor:

Jerrold Griggs

MATH 599 (SCHC 411)
Prerequisite: Grade of C or better in MATH 142

This is an experimental mathematics-appreciation course giving an introduction to some modern mathematical research areas that greatly contribute to the field of Data Science, an emerging research area about understanding data beyond its statistical interpretation. The problems of interest are the efficient extraction of reliable and quantifiable information from possibly corrupted data, fast processing of enormous amounts of data, and categorizing the phenomena behind a specific set of data.

The course is in the form of a *proseminar* introducing high-level theoretical mathematics through examples and basic models. Its goal is to spark interest in the featured areas by walking the students through the basic concepts rather than building a detailed technical scientific description. Different types of research data will be considered including data from terrain elevation, climate modeling, hyperspectral imaging, and electron microscopy used by the instructor and his collaborators in different projects.

Students with different backgrounds are encouraged to sign up for this course. The diversity of the participants helps the discussion by viewing the methods from different angles and areas of applications. The math majors can benefit from obtaining the basic information about topics of future theoretical courses. The science majors can learn different approaches that are used for processing and understanding scientific data. All will gain perspective on the methodology used in scientific software.

The course consists of seven modules:

- Basic Signal/Image Processing
- Fourier Transform and FFT
- Multiresolution and Wavelets
- Principle Component Analysis and Dimension Reduction
- Sparse Representations and Compressed Sensing
- Mathematical Learning Theory
- Uncertainty Quantification

Each module takes about two weeks starting with one or two introductory lectures followed by discussion and short (3 to 5 minute) individual presentations by the students on a chosen topic/example from the module.

Spring 2018 Professor:

Peter Binev