Pi Mu Epsilon
& Gamecock Math Club

Getting to Know Your 500-Level MATH Courses at the University of South Carolina

Event sponsored in part by

MATH 511/STAT 511: Probability

Prerequisite: Grade of C or higher or concurrent enrollment in MATH 241

The purpose of this course is to give you an introduction to probablity theory and probablity distributions. The material presented will not only serve as a basis for the subsequent courses, STAT 512/513, but is also extremely useful and fascinating in its own right. STAT 511 has a prerequisite of a standard multivariable calculus course, and a strong familiarity with differentitation, integration, infinite series and sequences, and related facts, is necessary. This course is very important for those of you considering careers in actuarial sciences.

The course covers the axiomatic approach to probability, counting techniques, Bayes Theorem, random variables, probability distributions for discrete and continuous random variables, mathematical expectation, moment generating functions, joint and conditional distributions for multiple random variables, and measures of association (covariance and correlation). This course focuses on both theory and application. You will be expected to derive theoretical results using algebra and calculus and apply these results to problems from a multitude of spplications.

Spring 2016 Professor:

George Androulakis

MATH 515 (STAT 523): Financial Mathematics II

Prerequisite: Grade of C or better in MATH 514 (STAT 522)

Complex mathematical techniques are now widely used in finance. This course is the second part of a two course sequence that provides an elementary introduction to these techniques and to the fundamental concepts of financial mathematics. We examine some mathematical models that are used in finance, e.g. to model risk and return of financial assets and to model the random nature of stock prices.

In the first part of the course we continue our investigation of the `No Arbitrage Principle'. A very important mathematical result known as the Separating Hyperplane Theorem will be proved in order to establish the `Fundamental Theorem of Asset Pricing' and the closely related `Arbitrage Theorem'. We also consider some other interesting mathematical consequences of the Separating Hyperplane Theorem such as the existence of a steady state vector for Markov matrices.

In the second part of the course we look at some mathematical problems arising in portfolio selection. Taking into account the investor's tolerance for risk leads to the problem of maximizing expected `utility'. Using the technique of `mean-variance analysis' we derive the main results of the famous Capital Asset Pricing Model which relates the riskiness of an individual stock to the overall market through a numerical coefficient known as `beta'. If time permits we will consider some discrete optimization problems such as the `knapsack problem' which arise in portfolio selection.

In the third part of the course we shall return to the problem of options pricing. Using the multi-period binomial model we shall find an algorithm for the pricing of the American put option. The existence of an `equivalent martingale measure' for which the discounted stock process is a `fair game' leads us naturally to the general theory of martingales. We shall examine some of the basic concepts and results of this theory such as the notion of a `stopping time' and Doob's Optional Sampling Theorem.

Spring 2016 Professor:

Lili Ju

MATH 520: Ordinary Differential Equations

Prerequisite: Grade of C or better in MATH 344 or MATH 544

Upon completion of this course, students will be knowledgeable about and will be able to analyze solutions to differential equations of the first order and linear systems of ordinary differential equations. They will also be able to apply these ideas to determine elementary qualitative properties of nonlinear systems.

Differential equations is the language of science. Many basic scientific laws express the change in one quantity in terms of the values of the other quantities. These laws can be combined to create a mathematical model for the physical situation. Once the model is found the challenge is to understand the "solution" to the model – often without actually having explicit formulas. The primary focus of this course is the mathematical analysis of differential equations. Students will learn a few special techniques to find analytic (but not necessarily explicit) solutions to differential equations.

Spring 2016 Professor:

Qi Wang

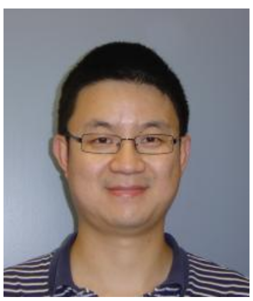
MATH 526: Numerical Linear Algebra

Prerequisite: Grade of C or higher in MATH 142

Linear algebra is the area of mathematics that looks at properties of systems of linear equations. In many realistic cases, these systems contain thousands, if not millions, of equations and unknowns. It is customary to formulate these problems in terms of matrices and vectors. This course is an introduction to the subject of linear algebra with attention given to numerical computations.

A linear system of equations in two dimensions corresponds to a collection of lines; in three dimensions to a collection of planes. Visualization in higher dimensions is not possible, but the same structure and general methods of analysis apply. Students will learn about properties of matrices, Gaussian elimination, ill-conditioned matrices, iterative solution methods, eigenvalue decomposition of a matrix, linear independence of vectors and over-determined systems.

Spring 2016 Professor:



Yi Sun

MATH 527 (CSCE 561): Numerical Analysis

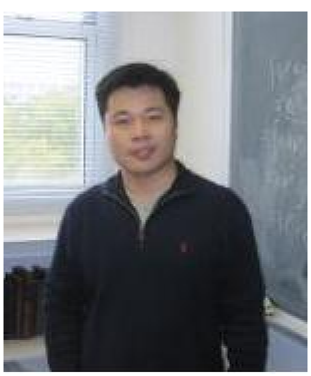
Prerequisite: Grade of C or better in MATH 520 or in both MATH 242 and MATH 344

Numerical Analysis studies the algorithms for the problems of continuous mathematics. The course will give an introduction to general ideas in Numerical Analysis and will discuss different aspects of the performance of the numerical procedures involved. In addition to the theoretical material, some numerical implementations in MATLAB will be considered on an elementary level. Topics include (not necessarily in the order they will be considered):

- number representations and loss of significance;
- polynomial interpolation;
- numerical differentiation;
- numerical integration;
- spline functions;
- method of least squares;
- numerical methods for ordinary differential equations;
- Monte Carlo methods.

At the end of this course students will be able to read, interpret, and use vocabulary, symbolism, and basic definitions from Numerical Analysis. The students will be able to use facts, formulas, and techniques learned in this course to apply algorithms and theorems to find numerical solutions and bounds on their errors to various types of problems including root finding, polynomial and spline approximation, numerical differentiation and integration, numerical solutions of ODEs.

Spring 2016 Professor:



Xinfeng Liu

MATH 532: Modern Geometry

Prerequisite: Grade of C or higher in MATH 300

The course focuses on two main topics: Non-Euclidean Geometry and modern approaches to Euclidean Geometry. The former topic will introduce the axioms of affine and projective planes and focus on establishing results from these axioms. As such, this part of the course will help build skills with proofs in mathematics. In addition, this part of the course will establish some concrete examples of affine and projective planes using modular arithmetic. The second part of the course will investigate the use of vectors, matrices, translations and rotations to establish interesting facts from Euclidean Geometry. The idea in this part of the course is to expose students to some fascinating material in Euclidean Geometry that goes beyond what one sees in a standard high school curriculum.

Spring 2016 Professor:

Michael Filaseta

MATH 544: Linear Algebra

Prerequisite: Grade of C or higher in MATH 300

Linear algebra is one of the fundamental topics in mathematics. Even if you do not know what linear algebra is, we have all been using many of the ideas for several years. While matrices will be common in this course, linear algebra is much more than “matrix algebra”. A second and equally important objective of this course is the exposure to mathematical proofs. The early parts of the course emphasize manipulative aspects more than theoretical issues. As the course progresses, however, the same topics will be revisited – with more of an emphasis on the abstract theory of linear algebra. Students will master concepts and solve problems based on matrix algebra, solution of linear systems, notions of vector space, linear independence, basis, and dimension, linear transformations, change of basis, eigenvalues, eigenvectors, and diagonalization.

A solid knowledge of linear algebra – both manipulations and theory – will be helpful in almost any upper-division course in mathematics or any course that uses mathematics: differential equations, numerical analysis, optimization, etc.

Spring 2016 Professors:

Andrew Kustin

Matthew Boylan

MATH 546: Algebraic Structures I

Prerequisite: Grade of C or higher in MATH 544

In this course, the student gets to experience mathematical thought beyond Calculus. As such, more sophistication is expected of the student. Most of the course will focus on group theory. Group theory is perhaps the area of mathematics with the fewest moving parts and the most ubiquity. Through studying group theory, each student will be exposed to the thought process involved in higher-level mathematics. Students will master concepts and solve problems based on permutation and abstract groups, subgroups, quotient groups, homomorphisms, isomorphisms, direct products, and rings.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.

Spring 2016 Professor:

Matthew Miller

MATH 547: Algebraic Structures II

Prerequisite: Grade of C or better in MATH 546

MATH 547 is the continuation of MATH 546. MATH 546 is about groups, while MATH 547 is about rings and fields. A field is a set F with two operations, usually called addition and multiplication. Under addition, F is an abelian group, with an identity element called 0. Under multiplication, $F\backslash\{0\}$, is an abelian group. The distributive axiom describes the interplay between the two operations. A ring is a set with two operations. Some of the field axioms hold in a ring. Some examples of fields are: the set of rational numbers, the set of real numbers, and the set of complex numbers. Every field is automatically a ring. The set of integers is a good example of a ring which is not a field. If R is a ring, then the set of all polynomials $\{f(x)\}$ with coefficients from R is another ring.

Students in this course will master concepts and solve problems on rings: ideals, polynomial rings, Euclidean domains, unique factorization domains; fields: extensions, Galois Theory, Euclidean constructions: modules over principal ideal domains.

Spring 2016 Professor:

Adela Vraciu

MATH 550: Vector Analysis

Prerequisite: Grade of C or higher in MATH 241

This course is an extension of MATH 241 and has much the same problem solving character with a dose of theory and abstraction. Proofs will be given in class and you will be expected to do some computationally driven proofs on your own, but the course will not have the same proof oriented character as a course like MATH 554. This course makes a nice transition from calculus to higher-level theoretical math courses.

After a quick review of MATH 241, the main material of the course consists of the "Big Three" Theorems of Green, Stokes, and Gauss and how to use them. Students will demonstrate an understanding of vector functions by solving problems in the context of vector fields (e.g., by distinguishing gradient fields from non-gradient fields), line integrals, surface integrals, divergence and curl. Students will be able to exploit algebraic and geometric methods to compute integrals using the theorems of Green, Stokes, and Gauss, as well as direct computation using parameterizations.

Spring 2016 Professors:

Daniel Dix

Paula Vasquez

MATH 552: Applied Complex Analysis

Prerequisite: Grade of C or higher in MATH 241

The emphasis of this course will be on the analysis of functions whose domain and/or range are sets of complex numbers. Much of this analysis will be very similar to the real-valued calculus that is the prerequisite for this course. Another objective is to define versions of elementary functions when the argument is a complex number. The "new" functions should be consistent with their real-valued counterparts and should maintain all of the usual properties.

The Cauchy Integral Theorem is one of the major triumphs of complex analysis. This theorem can be viewed as an extension of Green's Theorem (which provided a connection between double integrals and line integrals). One of the most important applications of the Cauchy Integral Theorem is the easy evaluation of many contour integrals.

Students will master the fundamental concepts from Complex Analysis, including the concept of a holomorphic function, complex line integrals, Cauchy's Theorem, Cauchy's Integral Formula, classification of zeros and singularities, and applications to residue calculus.

Spring 2016 Professor:

Ralph Howard

MATH 554: Analysis I

Prerequisite: Grade of C or higher in MATH 300 and at least one of MATH 511, 520, 534, 550, or 552

While most of science is based on inductive reasoning, mathematics is based on deductive reasoning. This means that new results are formed from logical combinations of hypothesis and statements accepted as true. Every result and technique learned in calculus (and other mathematics courses) is logically consistent and can be derived in a rigorous manner. In this course students begin to study some basic properties used to develop the fundamental calculus results including convergence of sequences, limit of a function, continuity (point-wise and uniform), derivative of a function, Rolle's theorem and the mean value theorem, L'Hospital's rule, inverse function theorem, Riemann integrals, Fundamental Theorem of Calculus, and derivatives of integrals. To be able to understand these results, and their proofs, it is necessary to develop a solid foundation in the real number system. It is also necessary to develop the ability to read, understand and write mathematical proofs. One of the most important steps in the creation of a mathematical proof is a solid understanding of the basic definitions. Unlike most previous courses you have taken, it is essential to pay attention to the details and technicalities. While this may be slightly unnatural, it is a skill that can be acquired through practice and patience.

Students will become knowledgeable about and will master concepts of real analysis. Students will improve their ability to write and read mathematical proofs, particularly those related to the least upper bound axiom, compactness, sequences, continuity, uniform continuity, differentiation, Riemann integration, and the Fundamental Theorem of Calculus.

This course is the first of a two-semester sequence. Both courses of the sequence are recommended for students planning to attend graduate school in mathematics.

Spring 2016 Professor:

Ryan Causey

MATH 555: Analysis II

Prerequisite: Grade of C or better in MATH 554

This course is a continuation of Math 554, Analysis I. If you enjoyed Math 554, you should strongly consider taking this course. It is also a must for anyone who is interested in pursuing graduate work in fields which use Mathematics. Topics which we will cover in this course include uniform continuity, derivatives, the Mean Value Theorem, L'Hopital's Rule, Riemann integrals, Riemann integrability, the Fundamental Theorem of Caluclus, integration by parts, sequences and series, pointwise convergence, uniform convergence, metric spaces, sets of measure zero, and a connection between the latter and Riemann integrability.

Spring 2016 Professor:

Maria Girardi

MATH 561: Introduction to Mathematical Logic

Prerequisite: Grade of C or better in MATH 300

From a formal point of view, mathematics can be construed as a collection of theorems, conjectures, proofs, counterexamples, examples, and definitions. Each of these can be seen as a string of symbols. Such strings are as amenable to mathematical investigation as are numbers, vectors, or functions. Roughly speaking this perception is at the heart of mathematical logic. In this setting, the notions of truth and proof can be given unambiguous meanings and their consequences can be developed as a separate branch of mathematics.

This course has the development of a basic but powerful part of this branch of mathematics at its heart. This part is called elementary or first-order logic---it is equipped with means of expression and proof rich enough to comprehend a considerable portion of mathematics.

Upon completion of the course, students should gain an understanding the concepts and the proofs of theorems takes precedence in this course over learning how to follow a set of directions to tackle a particular class of problems. Among the results of your efforts in this class should be a deeper understanding of both the power and the limitations of mathematics as well as an increased ability to discover and convey mathematical ideas and proofs.

This course will cover: Syntax and semantics of formal languages; sentential logic, proofs in first order logic; Godel's completeness theorem; compactness theorem and applications; cardinals and ordinals; the Lowenheim-Skolem-Tarski theorem; Beth's definability theorem; effectively computable functions; Godel's incompleteness theorem; undecidable theories.

Spring 2016 Professor:

George McNulty

MATH 574: Discrete Mathematics

Prerequisite: Grade of C or higher in MATH 300

Students will master concepts and solve problems in discrete mathematics, including propositional and predicate logic, basic set theory, counting, relations, and graphs. Students will master the fundamentals of proof techniques, including proof by contradiction, induction and the pigeon-hole principle. They will also master the concepts and be able to solve problems associated with enumeration, permutations and combinations, recurrence relations, and the groundwork for the more advanced topics of graph theory and game theory.

Spring 2016 Professor:

Joshua Cooper

MATH 575: Discrete Mathematics II

Prerequisite: Grade of C or better in MATH 574

Students will make progress with logical thinking, communicating mathematical ideas, and developing problem-solving skills, by

- writing up solutions to a wide range of homework exercises,
- learning and writing up proofs to sophisticated classical theorems on tests, and
- presenting solutions or projects to class.

We will prove several of the central discoveries of Graph Theory, learn statements of others that are too advanced to prove in this course, and think about complexity of algorithms for related problems. Among the topics we would like to cover this semester are:

1. Basic terminology and concepts for simple undirected graphs
2. Isomorphism, connectivity, degree sequences, spanning trees, cliques, independent sets
3. Euler and Hamilton paths and cycles
4. Extremal graph theory: Mantel's Theorem, Turan's Theorem
5. Matching theory: Hall's Theorem, Tutte's Theorem
6. Graph coloring: Brooks's Theorem
7. Lambda labellings: Delta-squared Conjecture
8. Planar graphs: Euler's formula, Kuratowski's Theorem, 5-Color Theorem, 4-Color Theorem
9. Trees, spanning tree

As time allows, students will be introduced to Directed graphs, multigraphs, hypergraphs, Ramsey Theory, Connectivity (Menger's Theorem), and Network flows

Spring 2016 Professor:

Jerrold Griggs

MATH 587 (CSCE 557): Introduction to Cryptography

Prerequisite: Grade of C or better in CSCE 145, or in MATH 241, and either CSCE 355 or MATH 574

This course should give you a well-rounded theoretical knowledge of the most important aspects of cryptography, together with sufficient practical mastery to permit implementation of some cryptosystems and cryptanalytic techniques in software.

Cryptography is essential to communicating securely through a public medium, such as the telephone or internet. Cryptography serves as the foundation for secure bank transactions, confidential e-commerce, and for military and national security-related communication, and is therefore ubiquitous in the wired and wireless world. Effective cryptography relies on several important mathematical as well as computer-scientific principles, and is thus well-suited for study (research and teaching) in both academic arenas. This is why the Math and CSE departments have decided to cross-list this course between them.

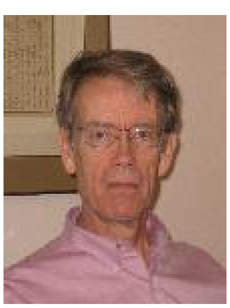
This course, together with a sequel graduate course in CSE in the implementation and use of methods for secure communication, is an essential component in the CSE department's certificate program in Information Assurance and Security. It also partially satisfies requirements for designating the Department of Computer Science and Engineering by the National Security Agency as a Center of Excellence in Information Assurance and Security.

This course covers the central topics in cryptography (the art of designing codes and ciphers), cryptanalysis (the art of breaking codes and ciphers), and cryptology (the mathematical science of cryptography and cryptanalysis).

Topics include: Design of codes and ciphers for secure communication, including encryption, authentication, and integrity verification: codes, ciphers, cryptographic hashing, and public key cryptosystems. Cryptological mathematical principles, cryptanalysis, and protocols for security.

We will introduce the requisite mathematical concepts when they are needed, including information theory, linear algebra, modular arithmetic, and arithmetic over finite fields.

Spring 2016 Professor:



Duncan A. Buell