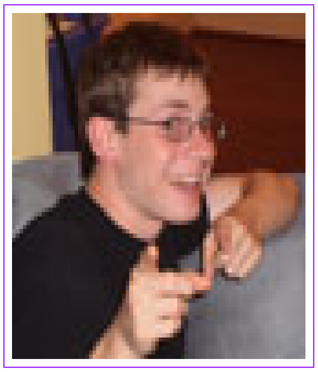
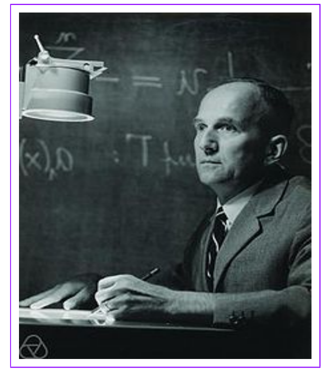
Pi Mu Epsilon
& Gamecock Math Club



Danny Rorabaugh

Student Seminar

Collatz Generalized : an expansion of the $3n+1$ problem



Lothar Collatz

Take the natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$. The Collatz "$3n+1$" function $C\colon \mathbb{N} \to \mathbb{N}$ is

$$C(n) \;=\; \begin{cases} 3n+1 & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even} . \end{cases}$$

Now take a look at the *trajectory* of a natural number n, which is just the infinite sequence (i.e., list)

$$\{\; n \;,\; C(n) \;,\; C(C(n)) \;,\; C(C(C(n))) \;,\; \ldots \;\} \;.$$

In 1937, Lothar Collatz conjectured that no matter what natural number $n \in \mathbb{N}$ you start with, the trajectory of n contains the number one. Sounds easy (at least to state).

The Collatz conjecture is still an unsolved problem although it has been verified to be true for $n < 10^{17}$.

In this seminar we will explore the related $An+B$ problem. Now define $C\colon \mathbb{N} \to \mathbb{N}$ by

$$C(n) \;=\; \begin{cases} An+B & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even} . \end{cases}$$

We will explore relationships between A, B, and n and whether a trajectory

- contains 1
- contains loops without reaching 1
- is unbounded with no positive integer occurring twice.

Understanding these relationships may help to shed light on the original unsolved Collatz conjecture.

Tue. 4 Oct. 2011
at 7pm in LC 310

Danny Rorabaugh is a mathematics graduate student at USC. This research was part of his undergraduate research project at Seattle Pacific University.

For more info visit us on FaceBook and at `http://www.math.sc.edu/~pme/`.