King Arthur's Problem

King Arthur had a problem. His daughter, Glissanda, loved mathematics so much that she spent most of her time solving problems, making geometric designs, and playing with numbers. That wasn’t King Arthur’s problem; he was proud of his daughter and her mathematical interest. Glissanda had reached the age when a young woman was permitted to marry, and she was definitely interested in marrying. In fact, she had one requirement for a husband: he must love mathematics (or at least like it a lot). For her, a life of evenings in front of a warm fire solving mathematical puzzles seemed like a sure way to marital bliss. Finding that mathematics-loving husband was King Arthur’s problem.

Now, if this story had taken place in modern times, King Arthur wouldn’t have had this problem. Glissanda would probably have met someone in her math classes who would be a fine mate, and that would be that. But in the days of King Arthur and his Knights of the Round Table, women didn’t have much freedom; their husbands were chosen by their parents.

Now King Arthur loved Glissanda dearly, and he would do anything for his daughter, but he was confused about how to find a husband to suit Glissanda. After all, the Knights of the Round Table—they were the best men in the land—were outdoor types who spent their time bravely scouting the countryside for dragons to slay. He couldn’t remember any of them ever even mentioning mathematics.
King Arthur was perplexed. He thought about it for days. And days stretched into weeks, but no ideas came to him.

Meanwhile he had his kingly work to do, but he became so distracted by this marriage problem that he couldn't concentrate. King Arthur was definitely not himself. One morning at the meeting he was very short-tempered with his knights. Little things seemed to bother him. At one point he even shouted out, "Can't you control that constant clanking of your armor and sit still?" The knights knew he must have something important on his mind.

That night at dinner King Arthur talked to Glissanda about the situation. "How shall I find out who is the cleverest in mathematics?" he asked her. "Should I just ask?"

"No, no," protested Glissanda. "That wouldn't be a good way. Some would answer yes just to become next in line to be king. I could get stuck with a husband who wants to do nothing at night except drink ale, one with no true interest in mathematical conversation. You must devise a mathematical test."

"What sort of test?" King Arthur asked.

"Let me think about it," Glissanda answered, wandering off, already deep in thought.
The next morning at breakfast Glissandra seemed cheerful.  
"Do you have the test?" her father asked.

"Not yet," Glissandra answered, "but I'm working on it. Tell me, father, how many knights are there at your Round Table?"

"Well, that varies," he replied. "It depends on how many are back from a journey. Sometimes as many as 50, and sometimes only a handful. Why?"

But Glissandra didn't answer. King Arthur could tell she was lost in thought, with that glaze over her eyes that told him she was thinking about mathematics again. It made him think that her husband would need to be very understanding.

King Arthur's face broke into a relieved smile. "Wonderful, wonderful!" he exclaimed. "But what if all the knights aren't present tomorrow? You know, I never can tell who will come."

"I've thought of that," Glissandra said. "In this test there is just one problem. Give it tomorrow to the knights who are present and announce that those interested in answering should reappear in one month's time with their solutions. In the meanwhile, they should spread the problem throughout the kingdom, so others who are off doing what knights do can come with solutions as well."

"What is the problem?" King Arthur asked eagerly.

That night at dinner Glissandra made an announcement. "I've got the test," she said. "You can give it at your meeting tomorrow to the Knights of the Round Table."
Glissanda explained, "Suppose 24 knights came to a meeting of the Round Table. And suppose the 24 chairs were numbered in order, so that everyone knew which chair was number 1, and in which direction you will count to 24. In order to choose my husband, you draw your sword, point to the knight in the first chair, and say, 'You live.' Then point to the knight in chair number 2, say, 'You die,' and chop off his head. To the third knight you say, 'You live.' And to the fourth, you say, 'You die,' and chop off his head. You continue doing this around and around the circle, chopping off the head of every other living knight until just one is left. That's the one I'll marry.

Glissanda stopped talking. "That's it?" her father asked, horrified. "You expect me to kill all of my knights but one? What kind of kingdom would I have then? There would be just you, your husband, a roomful of dead knights, and the rest of my knights cowering in the countryside for fear of ever returning to the Round Table. Is this what you call mathematics? Have you gone crazy?" King Arthur was shouting now. He couldn't believe his ears. He wanted Glissanda to be happy, but this was ridiculous.
"Oh, father," Glissanda said. "I wouldn't expect you to actually kill anyone. It's just a problem, and it definitely is mathematical. Besides," she went on, giggling a bit. "If you don't tell them you really won't chop heads, then only the brave knights will come. Then I'm sure to have a husband with courage as well as one with mathematical intelligence."

"But Glissanda," King Arthur went on, still rather upset, "I admit it's an unusual problem that is a true test of logical thought. But how do you know 24 knights will return that day to find the solution?"

Glissanda giggled a bit more, feeling even merrier. "That's the real point to the problem," she said. "Don't tell, but the knight of my dreams would know that he has solved the problem only if he knows where to sit for any number of chairs. I've been working on this problem, and there's a marvelous pattern for the solution!"

Which seat is the right one when there are 24 knights at the Round Table? Can you find the pattern for predicting which is the right seat for any number of chairs?