Some useful facts:

$\mathcal{L}\{0\} \equiv 0$

$\mathcal{L}\{t^n\} = n!/s^{n+1}$ for all non-negative integers n and $s > 0$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$ for $s > a$

$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$ for $s > 0$

$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}$ for $s > 0$

If $\mathcal{L}\{f(t)\} = F(s)$, then:
(a) $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$
(b) $\mathcal{L}\{\int_0^t f(\tau)\, d\tau\} = \frac{1}{s} F(s)$.
(c) $\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - \cdots - s f^{(n-2)}(0) - f^{(n-1}(0)$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

The general solution to a nonhomogeneous linear differential equation $L[y] = f(x)$ is $y_c + y_p$, where y_c is the general solution to the associated homogeneous equation $L[y] = 0$, and y_p is any solution to the original equation.

If the characteristic equation has the roots $r = s + iu$ and $r = s - iu$ then two linearly independent solutions to the original equation are $y_1(x) = e^s \cos(ux)$ and $y_2(x) = e^s \sin(ux)$.

$$\frac{dy}{dx} = g(x)h(y) = \frac{g(x)}{f(y)} \qquad \text{[Separable equation]}$$

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right) \quad \text{[homogeneous first order equation]} \quad \text{substitute } v = \frac{y}{x},\ y = vx$$

The word “homogeneous” means something completely different in second order equations.