Math 122 Makeup Notes

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1 Chapter 1

1.1 1.8 Function Composition

Function Composition: Suppose $f(t), g(t)$ are functions. The composite function $(g \circ f)(t) = g(f(t))$ (provided f is defined on the range of g). Here, we evaluate $f(t)$ first. Then we plug $f(t)$ as input into g.

Example: Suppose $f(t) = t + 1$ and $g(r) = \pi r^2$. Let's evaluate some compositions.

- Consider $f(g(3))$. We first evaluate $g(3) = \pi(3^2) = 9\pi$. Now we plug in 9π to f, to obtain $f(9\pi) = 9\pi + 1$, which is our final answer. So $f(g(3)) = 9\pi + 1$.
- Consider $g(f(2))$. Here, we evaluate $f(2)$ first. Note that $f(2) = 2 + 1 = 3$. Now we plug $f(2) = 3$ into g to obtain: $g(3) = 9\pi$. So $g(f(2)) = 9\pi$.
- We will often want to treat the composition of two functions *as a new function*, rather than finding a number. The approach remains the same as above. Consider $g(f(t))$. We plug $f(t)$ in, wherever we see r in $g(r)$. So $g(f(t)) = \pi(f(t))^2 = \pi(t + 1)^2$.

Example: Using the table, evaluate $g(f(0))$ and $f(g(0))$.

x	0	1	2	3
f(x)	3	1	-1	-3
g(x)	0	2	4	6

We have:

- Consider $g(f(0))$. The table tells us that $f(0) = 3$. So $g(f(0)) = g(3) = 6$.
- Consider $f(g(0))$. The table tells us that $g(0) = 0$. So $f(g(0)) = f(0) = 3$.

1.2 1.9- Proportionality

Direct/Inverse Proportionality: We say that y is directly (inversely) proportional to x is there exists a constant k s.t. $y = kx$ ($y = k/x$). Here, k is the constant of proportionality.

Ex: The heart mass of a mammal is proportional to its body mass.

(a) Suppose that a 70 kg human has a heart mass of 0.42 kg. Find the constant of proportionality. [$k = 0.42/70 = 0.006$.]

Answer: Unless stated otherwise, proportions are *direct.* So we have that $h(m) = km$, where m is the body mass and $h(m)$ is corresponding the heart mass. The problem tells us that a 70 kg human has a heart mass of 0.42 kg. So $0.42 = h(70) = 70k$. Solving for k, we have that: $k = 0.42/70 = 0.006$. Thus, $h(m) = 0.006m$.

(b) What is the heart mass of a 100 kg human?

Answer: From part (a), we have that $h(m) = 0.006m$. Thus, $h(100) = 0.006 * 100 = 0.6$ kg.

Inverse Proportionality: We say that y is inversely proportional to x is there exists a constant k s.t. $y = k/x$. Here, k is the constant of proportionality.

Ex: Four people can paint a fence in 3 hours. How long does it take 6 people to paint it?

- **Answer:** This problem deals with *inverse proportionality*. Why? Because with more people, the fence will get painted in less time. So $t(p) = k/p$, where p is the number of people and t is the time (in hours) to paint the fence. So the *larger* p gets, the smaller $t(p) = k/p$ gets. Now the problem statement tells us that: $t(3) = 4 = k/3$. Solving for k, we obtain that $k = 3 \cdot 4 = 12$. Thus, $t(p) = 12/p$. It follows that it takes 6 people $t(6) = 12/6 = 2$ hours to paint the fence.

Note: A variable can be proportional to a function:

- The period of a pendulum, T, is the amount of time required for the pendulum to make one complete swing. For small swings, the period T is proportional to $\sqrt{\ell}$, where ℓ is the pendulum's length. So $T = k\sqrt{\ell}$.
- An object's weight is inversely proportional to the square of its distance, r, from the earth's center. So:

$$w = \frac{k}{r^2}.$$

Power Function Proportionality: A function is said to be power function proportional if we may write $Q(x) = k \cdot x^p$, for some constant p.
Example: The following are examples of power functions:

- $y = 5/x^3 = 5x^{-3}$
- $y = 2/(3x) = \frac{2}{3}x^{-1}$
- $y = (5x^2)/2 = \frac{5}{2}x^2$.
- $y = 3\sqrt{x} = 3x^{1/2}$
- $y = (3x^2)^3 = 27x^6$

Example: Note that $y = 5 \cdot 2^x$ is *not* an example of a power function, as 2^x is an *exponential*.