MATH 550 Final Exam, Part B Name: ______________________
Spring, 2006

Note! For full credit you must show sufficient work to support your answer. In particular you must show the major steps of any integration. There are 60 points. Unless otherwise stated you may assume that curves, surfaces, and regions of $\mathbb{R}^3$ are suitable for application of the major theorems.

1. (6 points) Let $\mathbf{F}$ be defined on a domain $D$ in either $\mathbb{R}^2$ or $\mathbb{R}^3$. Give three different, but equivalent, statements that $\mathbf{F}$ be a conservative vector field, without mentioning curl.

2. (5 points) Suppose that $\mathbf{F}$ is everywhere tangent to the closed regular surface $S = \partial W$ for a suitable domain $W$ in $\mathbb{R}^3$. Demonstrate that $\iiint_W \text{div}(\mathbf{F})\,dV = 0$.

3. (8 points) Computing only derivatives (no integrals), explain why $\mathbf{G} = (3x^2z, \frac{2y}{1-y^2}, x^3)$ must be conservative in the slab $-1 < y < 1$, $-\infty < x < \infty$, $-\infty < z < \infty$.
4. (12 points) Compute the flux \( \iint_S \mathbf{F} \cdot d\mathbf{S} \) of \( \mathbf{F} = (xy^2, x^2y, z) \) through the closed cylinder \( S \) (oriented by outward normals) given by \( x^2 + y^2 = 4 \), \( z = -2 \), \( z = 3 \).

5. (18 points) In each of the following parts determine if \( \mathbf{F} \) is conservative or not, and give your reason. The compute \( \int_C \mathbf{F} \cdot ds \) by whatever method seems easiest and most appropriate.

a. \( \mathbf{F} = (z, 0, -x) \), \( C \) is the line segment from \((0, -1, 3)\) to \((1, 1, 4)\).
b. \( \mathbf{F} = (2xy, x^2 + 4y^3/(y^4 + z^4), 4z^3/(y^4 + z^4)), \) C is given by \( x = t, y = 4 \cos \theta, z = 3 \sin \theta, 0 \leq t \leq 1, 0 \leq \theta \leq \pi/2. \)

6. (5 points) We found for a vector field \( \mathbf{F} \) at a point \( P \), and an axis given by a unit vector \( \mathbf{n} \), that (curl \( \mathbf{F} \))(\( P \)) \( \cdot \mathbf{n} \) is equal to the “circulation” of \( \mathbf{F} \) about \( P \) on a surface with normal vector \( \mathbf{n} \). How is circulation defined?

7. (6 points) Using “flowboxes”, determine if each of the following vector fields has positive, negative, or zero divergence in general.