General instructions. Show your work. If you use major results quote them by name. Compute integrals in the easiest way that you can find; if your computation is turning out to be very complicated, think if a different approach might be possible. There are 120 points.

Stokes’ Theorem. Let \( S \) be a bounded, piecewise regular, oriented surface in \( \mathbb{R}^3 \) and suppose that \( C = \partial S \) consists of finitely many piecewise smooth simple closed curves, oriented consistently with the orientation of \( S \). Suppose that \( \mathbf{F} \) is a \( C^1 \) vector field defined on a domain that includes \( S \). Then

\[
\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} = \oint_C \mathbf{F} \cdot \mathbf{n} dS = \int_C \mathbf{F} \cdot ds.
\]

Recall that Green’s Theorem is a special case that applies when \( S \) lies in the plane and \( \mathbf{n} = \mathbf{k} \).

1. (9 points) Assume \( \mathbf{F} \) is a \( C^1 \) vector field in all of \( \mathbb{R}^3 \). Give three properties of \( \mathbf{F} \) if it happens to have the form \( \nabla f \).

2. (12 points) a. Show how to subdivide the following region \( D \) in the plane so that Green’s Theorem can be applied. Show how to orient \( \partial D \) so that the orienting normal vector is \(-\mathbf{k}\).

b. For the vector field \( \mathbf{F} \) shown below, give oriented paths \( C_1 \), \( C_2 \), and \( C_3 \) so that \( \int_{C_i} \mathbf{F} \cdot ds \) is positive, zero, and negative respectively.
3. (12 points) Let \( \Phi: D \to S \) be given by \( (x, y, z) = \Phi(u, v) = (uv^2, u^3, v) \) on \( D = [0,1] \times [0,1] \) (the unit square).
   a. Compute \( dS \) (vector) so that it (or \( \hat{n} \)) points upward.

   b. Compute \( dS \) (scalar).

4. (20 points) Let \( \mathbf{F} = (0, y/(y^2 + z^2), z/(y^2 + z^2)) \) and \( C \) be given by \( x = 0 \), \( y = 3 \cos t \), \( z = 2 \sin t \), \( 0 \leq t \leq \pi/2 \). Compute the work done by \( \mathbf{F} \), i.e., \( \int_C \mathbf{F} \cdot ds \).
5. (26 points) Compute \( \iiint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \) for \( \mathbf{F} = (3z^2 + yz, -y^2, x) \), and \( S \) the portion of the surface \( z = 10 - (x^2 + y^2) \) that lies above the plane \( z = 1 \) and is oriented so that \( \mathbf{n} \) points upward and away from the origin.

6. (15 points) Let \( \mathbf{F} = (z, 0, -x) \) and \( C \) be the line segment from \((1, 0, 2)\) to \((3,1,1)\). Compute \( \int_C \mathbf{F} \cdot d\mathbf{s} \).
7. (26 points) Compute the flux of \( \mathbf{F} \), i.e., \( \iint_S \mathbf{F} \cdot d\mathbf{S} \), through the closed cylinder \( S \) given by \( y = -1, \ x^2 + z^2 = 25 \), \( y = 2 \) and oriented by outward pointing normal vectors, where \( \mathbf{F} = (x, -y, z) \).