MATH 550 Fall, 2002 Exam #2 Name:______________________

Note! For full credit you must show sufficient work to support your answer. There are 100 points. Good luck!

1. (12 points) Let \( \mathbf{F} = x^2 y \mathbf{i} - (1/z) \mathbf{j} + \ln(y^2 + z^2) \mathbf{k} \).
   
   a. Compute the divergence of \( \mathbf{F} \).

   b. Compute the curl of \( \mathbf{F} \).

2. (8 points) On the vector field diagram below sketch the flowline \( \mathbf{r}(t) \) if \( \mathbf{r}(0) = (-5, 5) \), and the flowline if \( \mathbf{r}(0) = (-5, 4) \).
3. (4 points) High values of a function $h(x, y)$ are marked with H and low values with L in the contour plot shown below. Draw arrows to indicate what $\nabla h$ must look like in all parts of the plot.

4. (20 points) Let $w = \phi(x, y, z) = x^3z - y^2z^2$. Compute
   a. $\nabla \phi$
   
   b. an equation for the tangent plane to the level surface of $\phi$ that passes through $(-1, 1, 2)$.
   
   c. the directional derivative of $\phi$ at $(-1, 1, 2)$ in the direction of the vector $\mathbf{v} = (-6, 1, -2)$
   
   d. the maximum value of any directional derivative of $\phi$ at the point $(-1, 1, 2)$
5. (12 points) Let \( \mathbf{G} = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k} \). Find parametric equations for the flow line of \( \mathbf{G} \) that passes through the point \((2, 0, 1)\).

6. (5 points) Describe in words what is being shown in the diagrams below for the function \( f(x, y, z) = x^2 + y^2 - z^2 \). In particular if the two diagrams had been plotted in the same box, what would you see? Finally, which “sheet” in the box on the left corresponds to the highest value of \( c \) and which to the lowest?
7. (15 points) Re-express the integral \( \int_0^1 \int_{2y}^3 \cos(x^2) \, dx \, dy \) with the order of integration reversed, and compute the easier version.

8. (8 points) Set up completely (i.e., give the limits of integration determined by \( D \) and an explicit form of \( dV \), with everything in a consistent order), but do
NOT compute, a triple integral $\iiint_D xy e^{-z} \, dV$ for the region bounded by the $xy$-plane, the plane $z = x + 2$, and inside the cylinder $x^2 + 4y^2 = 4$.

9. (8 points) Set up completely (i.e., give the limits of integration determined by $D$ and an explicit form of $dV$, with everything in a consistent order), for a triple integral that gives the volume of the region $D$ bounded by $y = 16 - x^2 - z^2$ and $y = -2$. Do NOT compute.
10. (8 points) Let \( \mathbf{E}(x, y, z) = (f, g, h) \) be a vector field on \( \mathbb{R}^3 \), and assume that all partial derivatives are continuous. Show that \( \nabla \cdot (\nabla \times \mathbf{E}) = 0 \).