MATH 550    Fall, 2002    Exam #1    Name:____________________

Note! For full credit you must show sufficient work to support your answer. There are 100 points. Good luck!

1. (35 points) Let $\mathbf{A} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{B} = 7\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, and $\mathbf{C} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
   a. Find a unit vector $\mathbf{u}$ so that $\mathbf{u}$ is anti-parallel to $\mathbf{C}$ (parallel, but reversed in direction).

   b. Compute the projection of $\mathbf{A}$ along $\mathbf{B}$, namely $\text{proj}_B \mathbf{A}$.

   c. Find the standard equation of the plane $T$ that contains the points $P(1, 1, 0)$, $Q(5, -2, 1)$, and $R(8, -1, -3)$. Then give parametric equations for $T$. 
d. (continuation of problem 1) Find the volume of the parallelepiped with $A$, $B$, and $C$ as adjacent edges.

2. (20 points) Let $P(5, 3, -2)$ be a point; and $S$ be the plane $x - 7y - 5z = 6$.
   a. Give parametric equations for a line $L$ that passes through the point $P$ and that is perpendicular to the plane $S$.

   b. Give parametric equations for a line $M$ that passes through the point $P$ and that is parallel to the plane $S$.

   c. Find the distance from the point $P$ to the plane $S$. 
3. (20 points) a. Let \( L \) be the line given by \( \frac{x-3}{3} = \frac{y-3}{4} = \frac{z-1}{-1} \) and \( M \) be the line given by \[
\begin{align*}
x &= -t - 1 \\
y &= t \\
z &= 2t + 4
\end{align*}
\]. Do \( L \) and \( M \) intersect? If so, find the point(s) of intersection; if not explain why not.

b. If \( L \) and \( M \) do intersect, they lie in the same plane. If they do not intersect, then they lie in parallel planes, and one can be projected onto the plane of the other, so that the resulting two lines intersect, and have the same direction vectors as before (see diagram). Compute the angle between these two intersecting line. (Note: you do not need to do part a. in order to do part b.)
4. (15 points) Let $S$ be the surface given by the graph of the function $z = r^2$ for $1 \leq r \leq 4$. In the $xy$-plane sketch and label the level curves $C_{z=1}$, $C_{z=4}$, $C_{z=9}$, $C_{z=16}$, of the surface $S$. Sketch the surface $S$ in $\mathbb{R}^3$, and describe it in words, including a description of the vertical cross-sections (slices).

5. (10 points) Graph the surface whose spherical equation is $\rho = 1 - \sin \varphi$ (recall that in spherical coordinates $0 \leq \varphi \leq \pi$). The absence of $\theta$ in this equation gives insight into the symmetry of the graph: describe this symmetry in words.