MATH 241 Spring, 2010 Quiz #9 Name:____________________

For full credit you must show sufficient work that the method of obtaining your answer is clear.

1. Suppose P lies on the contour $f(x, y) = 10$ and Q lie on the contour $f(x, y) = 6$. Suppose C is a smooth curve from P to Q. Compute $\int_C \vec{\nabla} f \cdot d\mathbf{r}$.

2. Determine whether each of the following regions in the xy-plane is a simply connected region or not.
   a. $\{(x, y) \mid 1 \leq x^2 + y^2 \leq 4\}$ (an "annulus" or ring)

   b. $\{(x, y) \mid x < 0 \text{ if } y = 0\}$ (the plane with the origin and positive x-axis removed)

3. (6 points) A vector field $\mathbf{F}$ is shown below with three oriented curves. For each curve C_1, C_2, C_3 determine whether the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is positive, negative, or zero. No explanation is required.

(over $\rightarrow$)

4. Let $\mathbf{G} = \langle M, N \rangle = \langle y^2 + 2xy, x^2 + 2xy + \frac{1}{1+y^2} \rangle$, P be the point $(-1, 2)$, and Q be the point $(3, 1)$.

a. What is the domain of $\mathbf{G}$?

b. Explain why the integral $\int_P^Q \mathbf{G} \cdot d\mathbf{r}$ is independent of path.

c. Compute a potential function $g(x, y)$ for $\mathbf{G}$.

d. Evaluate by $\int_P^Q \mathbf{G} \cdot d\mathbf{r}$.