MATH 241 Spring, 2010 Exam #2 Name:____________________

For full credit you must show sufficient work that the method of obtaining your answer is clear. There are 100 points.

1. (15 points) Suppose $z = f(x, y)$ and you have the following information: $f(1, -2) = 5$, $f_x(1, -2) = -3$, $f_y(1, -2) = 1$.
   a. Estimate $f(0.9, -2.5)$ as accurately as possible using this information.

   b. Give an equation for the tangent plane to $z = f(x, y)$ at the point $(1, -2, 5)$.

2. (13 points) a. Match each contour diagram with its surface. Assume that adjacent contours in each diagram represent equal changes in the value of z.
   I- ______ , II- ______ , III- ______ , IV- ______ , V- ______ .

b. How can you be certain that (I) is NOT the contour diagram of a plane? (Hint: if $z = ax + by + c$, with a, b, and c constants, what are $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$?)

3. (35 points) Suppose $w = h(x, y, z) = z^2 + \ln(1 + xy)$, $x = s^3 t$, $y = s^2 \sin(st)$, and $z = 4 - t^2$.

a. Compute $\operatorname{grad} h = \vec{\nabla} h$ in terms of x, y and z.

b. Compute the directional derivative $D_{\hat{\mathbf{u}}} h(P)$ at the point $P(1, 3, -1/2)$ in the direction of $\mathbf{a} = \langle 2, 2, -1 \rangle$.

c. Give the maximum value for any directional derivative of h at P.

d. Which level surface for w (or h) is the point $Q(1, 1, -1)$ on? Give an equation for the tangent plane to this level surface at Q.

e. Compute $\dfrac{\partial w}{\partial s}$ in terms of x, y, z, s, and t, using the multivariable chain rule.

4. (25 points) Let $f(x,y) = xy^2 - 6x^2 - 3y^2$.

a. Find all relative (local) maximum, minimum and saddle points (and say which is which) that lie **inside** the circle $x^2 + y^2 = 64$. You may use the $D = f_{xx}f_{yy} - f_{xy}^2$.

b. SET UP the Lagrange multiplier equations (there are three of them) to find the maximum and minimum values of $f(x,y)$ along the boundary $x^2 + y^2 = 64$. DO NOT solve.

5. (12 points) The diagram shows level curves for the height of a surface $z = f(x, y)$ above (or below) the xy-plane.

a. If a small, smooth bead is gently placed on the surface over each point P, Q, and R, sketch the path it will roll, until it exits the scene (or state that it does not roll anywhere).

b. If the heavy curve represents a constraint $g(x, y) = 0$, explain why neither the absolute maximum nor minimum value of $z = f(x, y)$ on or inside the curve is found at R. Mark the point(s) S where these values might be found according to the method of Lagrange multipliers.

c. Box in a small portion of the diagram where it appears that the magnitude of the gradient vector of f is the largest. In which direction does it point?