1. (30 points) Let $P$ be the point $(-2, 1, 6)$; let $v = \langle 4, -1, -5 \rangle = 4\hat{i} - \hat{j} - 5\hat{k}$ and $w = \langle 2, 1, -1 \rangle$.
   
   a. Find the terminal point $Q$ of $v$ if the initial point is $P$.

   b. Compute the orthogonal projection of $v$ on $w$ (that is, $a = \text{proj}_w v$, the component of $v$ in the direction of $w$). Then compute the component of $v$ that is orthogonal to $w$; call this vector $b$. Finally, verify that $a$ is orthogonal to $b$.

   c. Give an equation for a plane whose normal vector is perpendicular to both $v$ and $w$, and which contains the point $P$. 
2. (20 points) A line $L_1$ passes through the points $(-3, -1, 3)$ and $(1, 1, -1)$. A line $L_2$ has equations

$$\frac{x + 3}{-1} = \frac{y - 4}{2} = \frac{z - 3}{1}.$$ 

a. Get a direction vector $\mathbf{v}$ for $L_1$ and then write the parametric equations for this line. Give a third point that is on $L_1$.

b. Is the point $(-2, 2, 4)$ on $L_2$? How can you tell?

c. These two lines DO intersect. Find the point of intersection.
3. (20 points) Give equations for a line L through the point P \((1,0,-2)\) that is \textbf{parallel} to the plane S: \(3x - y + 2z = 5\). Then find the distance from P to S.

4. (5 points) On the curve illustrated below, let \(\kappa_A\) and \(\kappa_B\) represent the curvatures at the points A and B, and let \(\rho_A\) and \(\rho_B\) represent the respective radii of curvature. Where is the curvature larger? Where is the radius of curvature larger? Show the osculating circles. Also show the unit normal vector \(\mathbf{N}\) at C.
5. (25 points) A particle moves so that \( \mathbf{r}(t) = (2 \cos t, 2 \sin t, t) \) for \( 0 \leq t \leq 2\pi \).

a. Compute the velocity vector \( \mathbf{v}(t) = \mathbf{r}'(t) \), the acceleration vector \( \mathbf{a}(t) = \mathbf{r}''(t) \), the speed \( \frac{ds}{dt} = ||\mathbf{v}(t)|| \), and the unit tangent vector \( \mathbf{T}(t) \).

b. Compute the arclength \( s \) of the path from time \( \tau = 0 \) to \( \tau = t \). How far has the particle traveled over the whole interval from 0 to \( 2\pi \)?

c. (Bonus) Recall that \( \mathbf{a}(t) = a_T \mathbf{T}(t) + a_N \mathbf{N}(t) \), where \( a_T = \frac{d^2s}{dt^2} = \frac{d}{dt} \left( \frac{ds}{dt} \right) \), \( a_N = \kappa \left( \frac{ds}{dt} \right)^2 \). Compute the curvature \( \kappa \) from one of the formulas \( ||\mathbf{T}'(t)||/||\mathbf{r}'(t)|| \), or \( ||\mathbf{r}'(t) \times \mathbf{r}''(t)||/||\mathbf{r}'(t)||^3 \), or by using \( ||\mathbf{a}(t)||^2 = a_T^2 + a_N^2 \).