MATH 241 Spring, 2010 Exam #1 Name:____________________

For full credit you must show sufficient work that the method of obtaining your answer is clear.

1. (30 points) Let P be the point $(-2, 1, 6)$; let $\mathbf{v} = \langle 4, -1, -5 \rangle = 4\hat{\mathbf{i}} - \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $\mathbf{w} = \langle 2, 1, -1 \rangle$.

   a. Find the terminal point Q of $\mathbf{v}$ if the initial point is P.

   b. Compute the orthogonal projection of $\mathbf{v}$ on $\mathbf{w}$ (that is, $\mathbf{a} = \text{proj}_{\mathbf{w}}\,\mathbf{v}$, the component of $\mathbf{v}$ in the direction of $\mathbf{w}$). Then compute the component of $\mathbf{v}$ that is orthogonal to $\mathbf{w}$; call this vector $\mathbf{b}$. Finally, verify that $\mathbf{a}$ is orthogonal to $\mathbf{b}$.

   c. Give an equation for a plane whose normal vector is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$, and which contains the point P.

2. (20 points) A line L_1 passes through the points $(-3, -1, 3)$ and $(1, 1, -1)$. A line L_2 has equations $\dfrac{x+3}{-1} = \dfrac{y-4}{2} = \dfrac{z-3}{1}$.

a. Get a direction vector $\mathbf{v}$ for L_1 and then write the parametric equations for this line. Give a third point that is on L_1.

b. Is the point $(-2, 2, 4)$ on L_2? How can you tell?

c. These two lines DO intersect. Find the point of intersection.

3. (20 points) Give equations for **a** line L through the point P $(1, 0, -2)$ that is **parallel** to the plane S: $3x - y + 2z = 5$. Then find the distance from P to S.

4. (5 points) On the curve illustrated below, let κ_A and κ_B represent the curvatures at the points A and B, and let ρ_A and ρ_B represent the respective radii of curvature. Where is the curvature larger? Where is the radius of curvature larger? Show the osculating circles. Also show the unit normal vector **N** at C.

5. (25 points) A particle moves so that $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, t\rangle$ for $0 \leq t \leq 2\pi$.

a. Compute the velocity vector $\mathbf{v}(t) = \mathbf{r}'(t)$, the acceleration vector $\mathbf{a}(t) = \mathbf{r}''(t)$, the speed $\frac{ds}{dt} = ||\mathbf{v}(t)||$, and the unit tangent vector $\mathbf{T}(t)$.

b. Compute the arclength s of the path from time $\tau = 0$ to $\tau = t$. How far has the particle traveled over the whole interval from 0 to 2π ?

c. (Bonus) Recall that $\mathbf{a}(t) = a_T\mathbf{T}(t) + a_N\mathbf{N}(t)$, where $a_T = \frac{d^2 s}{dt^2} = \frac{d}{dt}(\frac{ds}{dt})$, $a_N = \kappa(\frac{ds}{dt})^2$. Compute the curvature κ from one of the formulas $||\mathbf{T}'(t)||/||\mathbf{r}'(t)||$, or $||\mathbf{r}'(t) \times \mathbf{r}''(t)||/||\mathbf{r}'(t)||^3$, or by using $||\mathbf{a}(t)||^2 = a_T^2 + a_N^2$.