MATH 241 Spring, 1996 Exam #3 Name:______________________

Show your work for full credit. Calculators are allowed.

1. (20 points) Compute each double integral. Change the order of integration, if necessary.

   a. $\int_{\pi/6}^{\pi/2} \int_0^{\sin\theta} 6r\cos\theta \, dr \, d\theta$

   b. $\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3) \, dy \, dx$

2. (16 points) Let $\mathbf{F} = (yz + 4x)\mathbf{i} + xz\mathbf{j} + (xy - z^3)\mathbf{k}$.
   a. Compute $\text{div}(\mathbf{F})$.

   b. Compute $\text{curl}(\mathbf{F})$

   c. Is $\mathbf{F}$ conservative or not? Explain!

3. (10 points) Give plausible formulas for each of the vector fields described below.
   a. An force of attraction towards the origin in 3-space that weakens as the distance from the origin increases.

   b.

4. (8 points) Use diagram b. in the problem above. Give three paths, C_1 , C_2 , C_3 , composed just of line segments, from P to Q , so that $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} > 0$, $\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 0$, $\int_{C_3} \mathbf{F} \cdot d\mathbf{r} < 0$.

5. (12 points) Use Green's Theorem to compute $\oint_C y\,dx + xy\,dy$ where C is the boundary of the triangle with vertices $(0,0)$, $(3,0)$, and $(3,6)$, traversed counterclockwise.

6. (10 points) *Set up, but do NOT compute* the double integral that gives the volume of the solid in the first octant bounded by the surface $9z = 36 - 9x^2 - 4y^2$ and the coordinate planes.

7. (24 points) Compute each line integral, using the given parameterization.

a. $\int_C (2x + 9z)\, ds$, where C is the curve $x = t$, $y = t^2$, $z = t^3$, for $0 \leq t \leq 1$.

b. $\int_C \mathbf{G} \cdot d\mathbf{r}$, where $\mathbf{G} = (x + y, x - y)$ and C is the curve $x = \cos t$, $y = 3 \sin t$, for $0 \leq t \leq \pi/2$. Suggestion: recall that $\cos^2 t - \sin^2 t = \cos(2t)$.

c. Find a potential function for the vector field $\mathbf{G}$ of part b., and use this to re-compute the integral.