**MATH 241  Spring, 1996  Exam #2  Name:**____________________

*Show your work for full credit. Calculators are allowed.*

1. (10 points) Compute the partial derivatives $g_{xx}$ , $g_{xy}$ , and $g_{yy}$ for $g(x, y) = e^{-xy}$ .

2. (15 points) Let $f(r, s, t) = r^3 s - s^2 t^2$ and $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ .

   a. Compute the directional derivative of $f$ in the direction of $\mathbf{a}$ at the point $Q(-2, 1, 3)$ .

   b. What is the maximum value of any directional derivative of $f$ at $Q$ ?

   c. If, further, $r = y \sin x$ , $s = \arctan(xy)$ , and $t = \ln(x^2 + y^2)$ , compute $\frac{\partial f}{\partial x}$ .

3. (10 points) Let $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$. Show how $A$ transforms the unit square $\{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1\}$. What is the area of the transformed region? Is the transformation orientation-preserving or orientation-reversing? Show computations or give explanation!

4. (15 points) Let $w = 4x^2 + 3y^2 - 12z$ and $P$ be the point $(1, -1, 1/2)$.
   a. Sketch the level surfaces (and clearly label which is which!) $w = -12$, $w = 0$, and $w = 36$.

   b. The point $P$ is on what level surface for $w$? Give an equation for the tangent plane to that surface at the point $P$.

5. (12 points) Suppose $u = f(x - ct)$, where $c$ is a positive constant, and $f$ is a differentiable function.

a. Show that $u_{tt} = c^2 u_{xx}$, that is, $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

b. The graph $y = f(x)$ is shown below. Sketch the trace of the graph of $u$ for each of $t = -1, 1, 2$; label which graph is which. Then sketch the graph of $u$ as a function of $x$ and $t$ for $-2c \le x \le 3c$ and $-1 \le t \le 2$.

6. (13 points) Let $g(x,y) = x^3 + y^3 - 6xy$. Find all critical points for $g$ and indicate whether each is a local max, local min, or saddle point.

7. (12 points) The temperature on a circular disk $\{(x,y) | 0 \leq x^2 + y^2 \leq 1\}$ is $T = 2x^2 + y^2 - y$. Find the hottest and coldest spots on the disk.

8. (13 points) The formula $1/R = 1/R_1 + 1/R_2$ determines the combined resistance $R$ when resistors of resistance $R_1$ and $R_2$ are combined in parallel.

a. Suppose $R_1 = 100$ and $R_2 = 25$, each with a possible error in measurement of $0.5$. *Use differentials, or the microscope approximation,* to determine the maximum possible error in the computation of $R$.

b. *In general* what is the percent error in $R$ if there is a 2% error in the measurement of $R_1$, a 5% error in the measurement of $R_2$, and $R_1 = 4R_2$?