MATH 241 Spring, 1996 Exam \#1 Name:
Show your work for full credit. Calculators are allowed.

1. (22 points) Let $\mathbf{v}=4 \mathbf{i}+2 \mathbf{j}-\mathbf{k}, \mathbf{w}=3 \mathbf{i}-2 \mathbf{j}-3 \mathbf{k}$, and $P$ be the point $(-1,5,2)$. a. Compute $2 \mathbf{v}-\mathbf{w}$.
b. If $\mathbf{w}=\overrightarrow{P Q}$, compute the coordinates of the point $Q$.
c. Compute a unit vector in the opposite direction to $\mathbf{v}$.
d. Compute the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$.
e. Compute $\operatorname{pr}_{\mathbf{w}} \mathbf{v}$, the vector projection of $\mathbf{v}$ along $\mathbf{w}$.
f. Give parametric equations for the line that passes through the point $P$ and that has direction given by $\mathbf{v}$.
g. Give an equation for the plane whose normal vector is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$, and which contains the point $P$.
2. (10 points) A line $\ell_{1}$ passes through the points $(2,0,5)$ and $(1,3,1)$. A line $\ell_{2}$ has parametric equations $x=1+t, y=5+t, z=-4-6 t$.
a. Find parametric equations for $\ell_{1}$.
b. Find the point of intersection of the two lines. (Suggestion: use symmetric equations for one of the lines.)
3. (6 points) Find equations for a line $L$ that is parallel to the plane $2 x+4 y-z=5$, but does not lie in this plane.
4. (8 points) Compute $f_{r}$ and $f_{s}$ for $f(r, s)=r^{3} \ln \left(r^{6}+s^{2}\right)$.
5. (14 points) A particle moves so that $\mathbf{r}(t)=(5 t, 3 \cos t, 3 \sin t)$.
a. Compute the velocity, the speed, and the distance traveled from $t=0$ to $t=2 \pi$.
b. Describe the path of the motion from $t=0$ to $t=2 \pi$.
6. ( 8 points) Sketch the surface $x^{2}+y^{2}-4 z^{2}=16$. Show at least three different traces.
7. (4 points) Sketch the surface $4 x^{2}+8 z=0$.
8. (10 points) The density $\rho$ (in $\mathrm{g} / \mathrm{cm}^{3}$ ) of carbon dioxide gas is directly proportional to the pressure $P$ (in atmospheres) and inversely proportional to the temperature $T$ (in degrees Kelvin).
a. Write an equation for $\rho$ in terms of $P$ and $T$, and compute the proportionality constant if $\rho=.002$ at $T=273^{\circ} \mathrm{K}$ and $P=1.018$.
b. Compute $\frac{\partial \rho}{\partial P}$ and $\frac{\partial \rho}{\partial T}$.
9. (10 points) The table of values below gives heat index $I$ in terms of sample values of temperature $T$ and relative humidity $h$.
a. Is $I$ a linear function of $h$ and $T$ ? If yes, produce the formula; if no, explain why not.
b. Estimate $I$ if $T=88$ and $h=40$.
c. Estimate $I$ if $T=88$ and $h=44$.
10. (8 points) Give the number of the contour diagram that corresponds to each of the labeled 3D graphs.
A $\qquad$ B $\qquad$ C $\qquad$ D
