MATH 241Spring, 1996Exam #1Name:Show your work for full credit. Calculators are allowed.

- 1. (22 points) Let $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j} \mathbf{k}$, $\mathbf{w} = 3\mathbf{i} 2\mathbf{j} 3\mathbf{k}$, and P be the point (-1, 5, 2). a. Compute $2\mathbf{v} - \mathbf{w}$.
 - b. If $\mathbf{w} = \overrightarrow{PQ}$, compute the coordinates of the point Q.
 - c. Compute a unit vector in the opposite direction to $\, {\bf v} \, .$
 - d. Compute the cosine of the angle between \mathbf{v} and \mathbf{w} .

e. Compute $pr_{\mathbf{w}}\mathbf{v}$, the vector projection of \mathbf{v} along \mathbf{w} .

- f. Give parametric equations for the line that passes through the point P and that has direction given by \mathbf{v} .
- g. Give an equation for the plane whose normal vector is perpendicular to both \mathbf{v} and \mathbf{w} , and which contains the point P.

- 2. (10 points) A line ℓ_1 passes through the points (2,0,5) and (1,3,1). A line ℓ_2 has parametric equations x = 1 + t, y = 5 + t, z = -4 6t. a. Find parametric equations for ℓ_1 .
 - b. Find the point of intersection of the two lines. (Suggestion: use symmetric equations for one of the lines.)

3. (6 points) Find equations for a line L that is parallel to the plane 2x + 4y - z = 5, but does not lie in this plane.

4. (8 points) Compute f_r and f_s for $f(r,s) = r^3 \ln(r^6 + s^2)$.

- 5. (14 points) A particle moves so that $\mathbf{r}(t) = (5t, 3\cos t, 3\sin t)$.
 - a. Compute the velocity, the speed, and the distance traveled from t = 0 to $t = 2\pi$.

b. Describe the path of the motion from t = 0 to $t = 2\pi$.

6. (8 points) Sketch the surface $x^2 + y^2 - 4z^2 = 16$. Show at least three different traces.

7. (4 points) Sketch the surface $4x^2 + 8z = 0$.

- 8. (10 points) The density ρ (in g/cm³) of carbon dioxide gas is directly proportional to the pressure P (in atmospheres) and inversely proportional to the temperature T (in degrees Kelvin).
 - a. Write an equation for ρ in terms of P and T, and compute the proportionality constant if $\rho = .002$ at $T = 273^{\circ}$ K and P = 1.018.
 - b. Compute $\frac{\partial \rho}{\partial P}$ and $\frac{\partial \rho}{\partial T}$.

9. (10 points) The table of values below gives heat index I in terms of sample values of temperature T and relative humidity h.

- a. Is I a linear function of h and T? If yes, produce the formula; if no, explain why not.
- b. Estimate I if T = 88 and h = 40.
- c. Estimate I if T = 88 and h = 44.

10. (8 points) Give the number of the contour diagram that corresponds to each of the labeled 3D graphs. A _____ B ____ C ____ D ____