MATH 172 **Final Exam Part B** **Name:**____________________

Spring, 2004

This part of the exam covers the first 2/3 of the course material, and your score will replace the lower of yours scores on Exam 1 or Exam 2, provided this helps you. There are 100 points. For full credit you must show your work, providing sufficient evidence for your answers, which may be verbal, tabular, graphical, or other relevant presentation. Unless otherwise indicated **give your answers to at least 2 decimal place accuracy**.

1. (15 points) A population $A(t)$ of amoebae is growing **continuously** over time so that the **per capita** rate of increase is 0.04 g/day. (We don't count amoebae, we weigh them.)

   a. Write the model equation that describes this situation.

   b. If $A(0) = 5$ g, write the explicit solution equation, for this model equation. The initial population of 5 g grows to what size in 2 weeks (14 days)? If you can not find the explicit solution, approximate the answer by using Euler's method with $\Delta t = 2$ (and show your work on the last page).

   c. If instead of amoebae, A_n represents female antelope population, which reproduce only once a year, with a discrete growth rate of 0.04 antelope/year, and $A_0 = 50$ pregnant females, write the model equation. Then determine the female antelope population after $n = 10$ years.

2. (12 points) a. Formulate a model (dependent variable u, independent variable n) in which 80% of drug in the bloodstream from one day to the next is used up, but the remainder is reinforced by a maintenance dose of 48 mg/day.

   b. What is the theoretical steady state (equilibrium) amount of the drug in the bloodstream?

   c. Describe the short term pattern of the values of u_n if the initial dose is 24 mg.

3. (8 points) Shown below is the phase portrait of a two-variable system (R_n for rabbits, the prey; F_n for foxes, the predators) with the values of time in years. What are the initial conditions? ______________ ______________ Plot R_n and F_n against n from 0 to 10 in the space provided (and be sure to indicate which graph is which!). What appears to happen past time $n = 9$?

4. (5 points) Compute the equilibrium point (E, F) of the dynamical system

$$\begin{aligned} u_n &= 2u_{n-1} - v_{n-1} - 1 \\ v_n &= u_{n-1} - 3v_{n-1} + 8 \end{aligned}$$

5. (6 points) A mild infectious illness, spread by close contact at work or school, is making people sick (and infectious). Each week 12% of the healthy, but susceptible, population becomes sick, while 48% of the sick population recovers and is immune to getting re-infected. As the number of new sick people increases, they are spreading out over a larger area and more and more healthy people are put into a situation in which they might become infected; in fact the pool of healthy (but susceptible) people increases by 6 thousand each week. Write model equations for (in thousands) h_n, the number of healthy, but susceptible, people, s_n, the number of sick infectious people, and and r_n, the number of immune people.

6. (18 points) For each model equation, and initial condition, **first give the solution equation**. Then answer the other questions.

a. Model equation $u_n = (0.7)u_{n-1}$ with initial condition $u_0 = 50$; then describe the long term behavior of u_n.

b. Model equation $z_n = z_{n-1} + 1.3$ with initial condition $z_0 = 2.7$.

c. Model equation $v_n = (-0.3)v_{n-1} + 130$ with $v_0 = 200$. What is the long term behavior of v_n, and does the equilibrium appear to be stable? Explain.

d. In (c) compute the ratio $\dfrac{|v_n - E|}{|v_{n-1} - E|}$ and explain its significance by saying *how rapidly* v_n goes towards or away from the equilibrium.

7. (7 points) Determine the model equation satisfied by $Q(t) = a\cos 5t + b\sin 5t$. Suggestion: compute $Q'(t)$ and $Q''(t)$. Also determine the values of the constants a and b if $Q(0) = 8$ and $Q'(0) = 10$.

8. (7 points) Let $A = \begin{bmatrix} 2.3714 & 0.2571 \\ 0.3857 & 1.7286 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

a. Compute $A\mathbf{v}$ and $A\mathbf{w}$.

b. The eigenvalues for this matrix are $\lambda_1 = 2.5$ and $\lambda_2 = 1.6$, and eigenvectors are $\mathbf{v}$ and $\mathbf{w}$. Which goes with which? Briefly explain.

9. (9 points) Compute the sum of each series; or state that no sum exists, and why or why not.

   a. $\sum_{k=0}^{\infty}(-2/3)(15/6)^k$

   b. $\sum_{j=0}^{\infty}(15/6)(-2/3)^j$

10. (6 points) The period of $\sin(0.25x)$ is $x =$ ______ . Find A and B so that $A\cos(Bx)$ has an amplitude of 4 and a period of $x = 3\pi/2$.

11. (7 points) You are given annual survival probabilities and fecundities (numbers of surviving offspring) for a population with three stages: hatchlings, juveniles, and adults. From this you construct a population projection (Leslie-Lefkowitch) matrix A .

    a. How is $\mathbf{P}_{16}$ computed in terms of A and $\mathbf{P}_0$?

    b. You find that A has three eigenvalues, of which the dominant one is $\lambda = 1.03$ and the corresponding eigenvector is $\mathbf{v} = [\,2.57 \quad 0.90 \quad 0.23\,]^T$. What is the approximate long term annual growth rate? ______ In the long run what percent of the population are hatchlings? juveniles? adults? Or is there no stable age distribution? Explain.