MATH 172 **Spring, 2004** **Name:____________________**

Final Exam Part A

This part of the exam covers the last 1/3 of the course material, and essentially serves as Exam 3. There are 50 points. For full credit you must show your work, providing sufficient evidence for your answers, which may be verbal, tabular, graphical, or other relevant presentation. Unless otherwise indicated **give your answers to at least 2 decimal place accuracy**.

Recall that the affine second order dynamical system (model equation)

$$u_n = a_1 u_{n-1} + a_2 u_{n-2} + b$$

has an explicit general solution

$$u_n = c_1 r_1^n + c_2 r_2^n + E$$

where E is the equilibrium value, and r_1, r_2 are real and distinct solutions of the associated quadratic equation

$$x^2 = a_1 x + a_2 \qquad (*)$$

OR, if $(*)$ does not have real solutions, then the general solution is

$$u_n = c_1 r^n \cos(\theta n) + c_2 r^n \sin(\theta n) + E$$

where $re^{\theta i}$ is a complex solution of $(*)$. Note that if $(*)$ has a solution of form $h + ki$ (for example, $h = -2$, $k = 3$ comes from $(-4 \pm \sqrt{-36})/2 = -2 \pm (\sqrt{36}\, i)/2 = -2 \pm 3i$), then (r, θ) is simply the polar coordinate version of the point (h, k) in ordinary coordinates. [Of course, you DO KNOW the quadratic formula that $x = (-B \pm \sqrt{B^2 - 4AC})/(2A)$ if $Ax^2 + Bx + C = 0$!)]

1. (10 points) Suppose a dynamical system a solution

$$u_n = 9(0.3)^n \cos(1.0472\, n) + (0.3)^n \sin(1.0472\, n) + 20,$$

so we expect long term oscillatory behavior. Are the oscillations steady or damped or expanding, and how do you know? What is the period of these oscillations?

2. (20 points) Consider the dynamical system $u_n = 2.3u_{n-1} - 1.2u_{n-2} - 0.4$.

a. Find the equilibrium value E, if there is one.

b. Solve the associated quadratic equation for r_1 and r_2 or for r and θ (in radians).

c. Write the general solution (don't forget the $+E$ on the end). If $u_0 = 10$ and $u_1 = 9.5$, determine c_1 and c_2.

d. Describe the short and long term behavior of the system, using numerical evidence if necessary, and indicate how you can reconcile these behaviors with the solution found in part (c).

3. (10 points) Sketch, on the same graph, P as a function of t if $\frac{dP}{dt} = P' = rP(1 - \frac{P}{70})$. Label your graphs with a, b, c, d.
   a. if $r = 0.4$ and $P(0) = 35$
   b. if $r = 0.4$ and $P(0) = 5$
   c. if $r = 0.2$ and $P(0) = 90$
   d. if $r = 0.2$ and $P(0) = 5$

4. (10 points) The graph of the **per capita** growth rate $g(P)$ is given below.
   a. Write $P'(t)$ in terms of $g(P)$ and P.

   b. At which value(s) of P is $P' = 0$? What is the significance of these values of P?

   c. At which values of P is $P' > 0$? What is the significance of these values of P?

   d. If $P(0) = 8$, what will happen to this population? In real life terms, why might this be plausible biologically?