MATH 172 Spring, 2004 Exam #2 Name:____________________

There are 100 points. **For full credit you must show your work**. You may use a calculator, but this does not exempt you from explaining your answers by giving results of computations or sketches of graphs, etc.

1. (36 points) For each model equation, and initial condition, **first give the solution equation**. Then answer the other questions.

   a. Model equation $u_n = (1.07)u_{n-1}$ with initial condition $u_0 = 350$; then $u_5 =$ ______ .

   b. Model equation $P'(t) = 0.07P(t)$ with initial contion $P(0) = 350$; then $P(5) =$ ______ . The population is double the initial population when $t =$ ______ .

   c. Model equation $z_n = z_{n-1} + (3/2)$ with initial condition $z_0 = 9$.

   d. Model equation $v_n = 0.96v_{n-1} + 5$ with $v_0 = 200$. Does the equilibrium appear to be stable (yes or no)? ______ . **Briefly** explain.

   e. In (d) compute the ratio $\dfrac{v_n - E}{v_{n-1} - E}$ and explain its significance by saying *how rapidly* v_n goes towards or away from the equilibrium.

f. (7 bonus points) Model equation $P' = -0.04P + 5$ with $P(0) = 200$.

2. (10 points) Verify that $Q(x) = 2x^2 + cx + d$, where c and d are constants, satisfies the model equation $Q''(x) = 4$. Compute the values of c and d so that $Q(0) = 5$ and $Q'(0) = 3$.

3. (7 points) Convert $r = 2$, $\theta = 5\pi/6$ (radians) to (x, y) coordinates. Also give the equivalent measure of $\theta = 5\pi/6$ in degrees.

4. (10 points) The period of $\sin(3x)$ is $x =$ ______ . Find A and B so that $A\cos(Bx)$ has an amplitude of 5 and a period of 4.

5. (12 points) Determine the model equation satisfied by $R(t) = -3\cos 5t + 2\sin 5t$.

6. (15 points) Let $A = \begin{bmatrix} 1.3 & -0.2 \\ 0.15 & 0.9 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

a. Compute $A\mathbf{v}$ and $A\mathbf{w}$.

b. The eigenvalues for this matrix are $\lambda_1 = 1.2$ and $\lambda_2 = 1$, and eigenvectors are $\mathbf{v}$ and $\mathbf{w}$. Which goes with which? Briefly explain.

c. Find a vector that lines up with the eigenvector belonging to eigenvalue 1, but whose column total is 1.

7. (10 points) Compute the sum of each series; or state that no sum exists.

   a. $\sum_{k=0}^{\infty}(3/10)(-1/5)^k$

   b. $\sum_{j=0}^{\infty}(1/4)(5/2)^j$

8. (6 bonus points) In a 2-variable system you find that eventually $u_n \approx 1.09u_{n-1}$ and $v_n \approx 1.02v_{n-1}$. Is the total population $T_n = u_n + v_n$ growing or decling? Can you say at what rate? Is there eventually a stable distribution? Briefly explain your answers.

9. (6 bonus points) Given below is the transition matrix for a weather model with three states: sunny (S), cloudy (C) and rainy (R).

$$A = \begin{array}{cccc} \text{tomorrow}\downarrow \backslash \text{today}\rightarrow & S & C & R \\ S & 1/2 & 0 & 1/8 \\ C & 1/4 & 1/4 & 1/8 \\ R & 1/4 & 3/4 & 3/4 \end{array} \qquad \mathbf{v} = \begin{bmatrix} 1/6 \\ 1/6 \\ 2/3 \end{bmatrix}.$$

If it is rainy today, what is the probability that it is cloudy tomorrow? _______ .
If it is rainy today, what is the probability that it is cloudy the day after tomorrow? _______ (Hint: don't do more work that is absolutely necessary!) One can show that $A\mathbf{v} = \mathbf{v}$; what is the significance of this observation?