MATH 172 Spring, 2002 Exam #2 Name:____________________

There are 100 points. For full credit you must show your work. If you use your calculator for anything more than simple arithmetic, say so!

1. (28 points) Miscellaneous questions involving trigonometric functions.

a. Convert $r = 2$, $\theta = 5\pi/6$ (radians) to (x, y) coordinates. Also give the equivalent measure of $\theta = 5\pi/6$ in degrees.

b. We have points $P(1, 1, -1)$, $Q(0, 2, 1)$, and $R(-1, 1, -2)$ and vectors that run from one to another: $\mathbf{v} = \overrightarrow{PQ}$ and $\mathbf{w} = \overrightarrow{PR}$. Find the angle between $\mathbf{v}$ and $\mathbf{w}$ (you may use the formula $\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}||\mathbf{w}| \cos\theta$).

c. Compute the derivative of $g(t) = t^3 \cos(\frac{1}{2}t) + \sin(t^6 + 1)$.

d. Give a function $h(x)$ that has period π, amplitude 5, and $h(\pi/4) = -5$.

2. (20 points) Let $\mathbf{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}$.

a. Compute $\mathbf{w} - 2\mathbf{v}$, $|\mathbf{v}|$, and $\mathbf{v} \cdot \mathbf{w}$.

b. Is $\mathbf{v}$ an eigenvector for A? If so, what is the eigenvalue; if not, why not?

c. Is $\mathbf{w}$ an eigenvector for B? If so, what is the eigenvalue; if not, why not?

3. (12 points) Given below is the transition matrix for a weather model with three states: sunny (S), cloudy (C) and rainy (R).

$$A = \begin{array}{cccc} \text{tomorrow}\backslash\text{today} & S & C & R \\ S & 1/2 & 0 & 1/8 \\ C & 1/4 & 1/4 & 1/8 \\ R & 1/4 & 3/4 & 3/4 \end{array} \qquad \mathbf{v} = \begin{bmatrix} 1/6 \\ 1/6 \\ 2/3 \end{bmatrix}.$$

If it is rainy today, what is the probability that it is cloudy tomorrow? ______ If it is rainy today, what is the probability that it is cloudy the day after tomorrow? ______ (Hint: don't do more work that is absolutely necessary!) What is the significance of the vector $\mathbf{v}$?

4. (30 points) You are given annual survival probabilities and fecundities (numbers of surviving offspring) for a population with three stages: hatchlings, juveniles, and adults.

a. Construct the transition matrix A.

$$\begin{array}{ccc} H_t & & H_{t+1} \\ J_t & \xrightarrow{80\%} & J_{t+1} \\ A_t & \xrightarrow[90\%]{} & A_{t+1} \end{array}$$

a. The initial population vector is $\mathbf{P}_0 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{P}_{31} = \begin{bmatrix} 368 \\ 119 \\ 28 \end{bmatrix}$. What is the distribution of the population into the various stages after 31 years? How is $\mathbf{P}_{31}$ computed in terms of A and $\mathbf{P}_0$?

b. You also find that A has three eigenvalues λ_i and corresponding eigenvectors $\mathbf{v}_i$, and that $\mathbf{P}_0$ can be written in terms of these.

$$\lambda_1 = -0.176 \qquad \lambda_2 = 0.767 \qquad \lambda_3 = 1.109$$

$$\mathbf{v}_1 = \begin{bmatrix} 6.629 \\ -0.679 \\ 0.032 \end{bmatrix} \qquad \mathbf{v}_2 = \begin{bmatrix} 0.183 \\ -0.551 \\ 0.207 \end{bmatrix} \qquad \mathbf{v}_3 = \begin{bmatrix} 1.194 \\ 0.386 \\ 0.092 \end{bmatrix}$$

$$\mathbf{P}_0 = 13.071\mathbf{v}_1 - 7.472\mathbf{v}_2 + 12.326\mathbf{v}_3$$

c. Get an exact formula for $\mathbf{P}_t$. You may leave the $\mathbf{v}_i$'s in your answer.

d. Give a formula for $\mathbf{P}_t$ that approximates it well when t is large.

e. What is the approximate annual growth rate? _______

f. In the long run what growth stage forms the majority of the population, and how can you tell?