

**MATH 172X Spring, 2001 Exam #2 Name: \_\_\_\_\_**

There are 100 points. For full credit you must show your work. If you use your calculator for anything more than simple arithmetic, say so!

1. (8 points) Convert  $r = 3$ ,  $\theta = \pi/6$  (radians) to  $(x, y)$  coordinates. Also give the equivalent measure of  $\theta = \pi/6$  in degrees.
  
2. (10 points) What is the amplitude and period of  $y = -2\cos(4x)$ ? What effect does the minus sign have on the graph of this function compared to  $y = 2\cos(4x)$ ?
  
3. (12 points) Compute the derivative of  $P(t) = 7\cos(2t) - \sin(t^4 + 1)$ .
  
4. (12 points) On a typical MWF morning, I find it hard to get to my 9:05 class. The chance that I get breakfast is 0.2; the chance that I get a cappuccino (coffee) is 0.4; the chance that I get both is 0.1.
  - a. What is the probability that I get breakfast or coffee or both?
  
  - b. If I do not get coffee, which has probability \_\_\_\_\_, then the probability that I am grumpy is 0.8. If I do get coffee, then for sure I will not be grumpy. What is the probability that I am grumpy?

5. (12 points) Compute the antiderivative:

a.  $\int 6x^2 \sin(x^3 + 1) dx$  .

b.  $\int \sin^4 x \cos x dx = \int (\sin x)^4 \cos x dx$  .

6. (5 points) Compute  $\binom{11}{6}$  by hand, showing all the arithmetic and cancellations.  
(You may use your calculator to check!)

7. (41 points) In this problem you can leave symbols like  $\binom{n}{k}$ ,  ${}_nC_k$ ,  ${}_nP_k$ ,  $n!$ ,  $66 \cdot 65 \cdot 64$ ,  $5^{10}$ , etc. in your final answer. The top shelf of your bookcase holds 24 books. You have 4 books on mathematical biology (M), 8 on ecology (E), 6 on cell biology and genetics (C), 5 on evolution and the Darwinian theory of natural selection (D), and one John Grisham thriller (T) that you picked up in the airport when you had a long lay-over. The parts of this problem do not depend on one another.
- Scanning the shelf from left to right, how many possible arrangements of these books are there?
  - If all you care about the 24 books is what topic is where, how many possible arrangements of these books are there? (Hint: think about positions on the shelf.)
  - Suppose all you care about is to group the books by topic, but the topics can be in any order, and within each clump you don't care what the order is. How many arrangements are possible?
  - Your actual collection is much larger: 5 M's, 12 E's, 15 C's, 10 D's and 4 T's. In how many ways can you select from your entire collection to fill the top shelf as stated in the beginning of the problem?
  - Your collection is described in part d. You load the top shelf either by using the M's, C's, and T's, or by using the D's and E's with 2 spaces left over at the far right end for future acquisitions. How many possible arrangements are there if you group the books by topic, and you do NOT care about the order of the books within each topic? (Hint: after you have chosen which types to use, consider how many ways the topics can be arranged.)