MATH 172 Spring, 2005 Exam #1 Name:____________________

There are 100 points. **For full credit you must show your work**. All answers should be **correctly rounded to 2 decimal places**, unless more are requested.

1. (25 points) a. Formulate a discrete model (dependent variable u, independent variable n) in which 80% of drug in the bloodstream from one day to the next is used up, but the remainder is reinforced by a maintenance dose of 40 mg/day. The initial dose is 10 mg.

   b. What is the equilibrium (steady state) amount of the drug in the bloodstream?

   c. Compute u_1, u_2, u_3, u_5, u_{10}.

   d. Describe the long term behavior of u_n if the initial dose is instead $u_0 = 100$ mg. Does u_n increase, decrease, oscillate, tend towards or away from the equilibrium?

   e. Does the equilibrium value you found in part (b) appear to be stable or unstable? Explain verbally and / or graphically, using your answers to (c) and (d).

2. (20 pts) A population $D(t)$ of fruitflies (*Drosophila*) is growing **continuously** so that the **per capita** rate of increase is 0.04 /day.

a. Write the model equation that describes this situation.

b. If $D(0) = 100$, write the explicit solution for this model equation. How many flies are present after 12 days?

c. Suppose you don't have a way to compute exponential functions. Show how to get an **approximation** for the fly population in 12 days by using three steps. In this case $\Delta t =$ ______ .

step n	time t	$D(t)$	$D'(t)$	$\Delta D \approx D' \Delta t$	$D(t + \Delta t)$

d. Instead of fruitflies, D_t represents female deer in a near-urban population, where there are no predators. We use a discrete growth model since deer reproduce only once a year; assume the discrete growth rate is 4%. Write the model equation for this situation and determine D_{25} if $D_0 = 100$.

3. (8 pts) Given vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and a matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$, show how A transforms each of the vectors $\mathbf{v}_i$ for $i = 1, 2, 3$, that is, compute $\mathbf{w}_i = A\mathbf{v}_i$ for $i = 1, 2, 3$. Use this to show how A transforms the unit square.

4. (15 points) The length of a steel and concrete bridge span ℓ depends on the temperature t in degrees Fahrenheit. For a rise in temperature of $1°F$, the length of the span increases by 0.012 feet. At $78\,°F$, the bridge is exactly 1000 feet long. In this problem keep 3 decimal place accuracy.

   a. Let's suppose we can only measure temperature to the nearest whole degree. Which type of model is more appropriate: discrete or continuous?

   b. Write the discrete model equation for this situation.

   c. Write the explicit solution for this model, i.e., give ℓ as a function of t.

5. (12 points) A chemical reaction involves constituents A, B and C. Each minute 15% of A is converted into B, while 25% of B converts naturally back into A. Meanwhile 15% of B converts into C. To keep the system running 3 units of B are added each minute. Write model equations for a_n, the amount of A, b_n, the amount of B, and c_n, the amount of C at time n minutes.

b. Without doing any calculations, explain why the amount of C must be steadily increasing if there is any A in the system at all.

6. (7 points) Compute the equilibrium point (E, F) of the dynamical system

$$\begin{aligned} u_n &= 2u_{n-1} - 2v_{n-1} + 4 \\ v_n &= -3u_{n-1} + 4v_{n-1} + 9 \end{aligned}$$

7. (15 points) Here is a table of values for a 2-variable dynamical system

n	0	1	2	3	4	5	6	7	8
u_n	14	10	7	8	12	14	10	9	11
v_n	13	12	5	2	4	8	7	5	6

Plot u_n and v_n against one another on one graph, and label the points with the values of n from 0 to 8. Plot u_n and v_n on a single graph against n from 0 to 8. If you were told that this system has an equilibrium point $(10, 6)$, would you say this equilibrium is stable or unstable? Why?

Extra workspace