1. (25 points) a. Formulate a discrete model (dependent variable $u$, independent variable $n$) in which 80% of drug in the bloodstream from one day to the next is used up, but the remainder is reinforced by a maintenance dose of 40 mg/day. The initial dose is 10 mg.

b. What is the equilibrium (steady state) amount of the drug in the bloodstream?

c. Compute $u_1$, $u_2$, $u_3$, $u_5$, $u_{10}$.

d. Describe the long term behavior of $u_n$ if the initial dose is instead $u_0 = 100$ mg. Does $u_n$ increase, decrease, oscillate, tend towards or away from the equilibrium?

e. Does the equilibrium value you found in part (b) appear to be stable or unstable? Explain verbally and / or graphically, using your answers to (c) and (d).
2. (20 pts) A population \( D(t) \) of fruitflies (\textit{Drosophila}) is growing \textbf{continuously} so that the \textbf{per capita} rate of increase is \( 0.04 \) /day.

a. Write the model equation that describes this situation.

b. If \( D(0) = 100 \), write the explicit solution for this model equation. How many flies are present after 12 days?

c. Suppose you don’t have a way to compute exponential functions. Show how to get an \textbf{approximation} for the fly population in 12 days by using three steps. In this case \( \Delta t = \ldots \).

\[
\begin{array}{ccccc}
\text{step} & \text{time} & \text{D(t)} & \text{D’(t)} & \Delta D \approx D’ \Delta t & D(t + \Delta t) \\
\end{array}
\]

d. Instead of fruitflies, \( D_t \) represents female deer in a near-urban population, where there are no predators. We use a discrete growth model since deer reproduce only once a year; assume the discrete growth rate is \( 4\% \). Write the model equation for this situation and determine \( D_{25} \) if \( D_0 = 100 \).
3. (8 pts) Given vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and a matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$, show how $A$ transforms each of the vectors $\mathbf{v}_i$ for $i = 1, 2, 3$, that is, compute $\mathbf{w}_i = A\mathbf{v}_i$ for $i = 1, 2, 3$. Use this to show how $A$ transforms the unit square.

4. (15 points) The length of a steel and concrete bridge span $\ell$ depends on the temperature $t$ in degrees Fahrenheit. For a rise in temperature of $1^\circ F$, the length of the span increases by 0.012 feet. At $78^\circ F$, the bridge is exactly 1000 feet long. In this problem keep 3 decimal place accuracy.
   a. Let's suppose we can only measure temperature to the nearest whole degree. Which type of model is more appropriate: discrete or continuous?
   b. Write the discrete model equation for this situation.
   c. Write the explicit solution for this model, i.e., give $\ell$ as a function of $t$. 
5. (12 points) A chemical reaction involves constituents A, B and C. Each minute 15% of A is converted into B, while 25% of B converts naturally back into A. Meanwhile 15% of B converts into C. To keep the system running 3 units of B are added each minute. Write model equations for $a_n$, the amount of A, $b_n$, the amount of B, and $c_n$, the amount of C at time $n$ minutes.

b. Without doing any calculations, explain why the amount of C must be steadily increasing if there is any A in the system at all.

6. (7 points) Compute the equilibrium point $(E,F)$ of the dynamical system

\[
\begin{align*}
    u_n &= 2u_{n-1} - 2v_{n-1} + 4 \\
    v_n &= -3u_{n-1} + 4v_{n-1} + 9
\end{align*}
\]
7. (15 points) Here is a table of values for a 2-variable dynamical system

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_n$</td>
<td>14</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>12</td>
<td>14</td>
<td>10</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>$v_n$</td>
<td>13</td>
<td>12</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>7</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Plot $u_n$ and $v_n$ against one another on one graph, and label the points with the values of $n$ from 0 to 8. Plot $u_n$ and $v_n$ on a single graph against $n$ from 0 to 8. If you were told that this system has an equilibrium point $(10, 6)$, would you say this equilibrium is stable or unstable? Why?

Extra workspace