MATH 172  Spring, 2004  Exam #1  Name:____________________

There are 100 points. For full credit you must show your work. You may use a calculator, but this does not exempt you from explaining your answers by giving results of computations or sketches of graphs, etc.

1. (25 points) a. Formulate a model (dependent variable $u$, independent variable $n$) in which 60% of drug in the bloodstream from one day to the next is used up, but the remainder is reinforced by a maintenance dose of 60 mg/day. The initial dose is 70 mg.

b. What is the steady state amount of the drug in the bloodstream?

c. Compute $u_1$, $u_2$, $u_3$, $u_5$, $u_{10}$.

d. Describe the long term behavior of $u_n$ if the initial dose is instead $u_0 = 125$ mg. Does $u_n$ increase, decrease, oscillate, tend towards or away from the equilibrium?

e. Is the equilibrium value you found in part (b) stable or unstable? Explain verbally and / or graphically.
2. (20 points) A population \( B(t) \) of bacteria is growing \textit{continuously} over time so that the \textit{per capita} rate of increase is 0.071/day. (We don’t count bacteria, we weigh them.)

a. Write the model equation that describes this situation.

b. If \( B(0) = 5 \) g (or 5000 mg), write the solution equation, for this model equation. This initial population of 5 g grows to what size in 31 days?

c. If instead of bacteria, \( B_t \) represented female bison population, which reproduce only once a year, with a discrete growth rate of 0.071/year, determine \( B_{31} \) if \( B_0 = 5 \).


3. (20 points) A chemical reaction involves constituents A, B, and C, in which C acts as an enzyme catalyzing the conversion of A into B. Each minute 45% of A is converted into B, while 10% of B converts naturally back into A. Meanwhile 5% of the enzyme C is used up. To keep the system running 25 units of A and 10 units of C are added each minute. Write model equations for \( a_n \), the amount of A, \( b_n \), the amount of B, and \( c_n \), the amount of C, at time \( n \) minutes.
4. (10 points) Give the “model equation” ("dynamic approach") and a “solution equation” ("explicit approach") for the height $h_n$ of a stack of $n$ chairs, where each chair is 3 feet high, and when you stack a new chair onto the pile, only 8 inches sticks out above the previous chair. Note that the pattern doesn’t really begin until you actually have one chair, so $h_0$ is not defined and $h_1 = 3$. Be clear and consistent with your units!

5. (15 points) Here is a table of values for a 2-variable dynamical system

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_n$</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>$v_n$</td>
<td>4</td>
<td>8</td>
<td>9</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>9</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Plot $u_n$ and $v_n$ against one another on one graph, and label the points with the values of $n$ from 0 to 9. Plot $u_n$ and $v_n$ on a single graph against $n$ from 0 to 8. What conclusion do you draw about this dynamical system, at least with the initial condition $u_0 = 3$ and $v_0 = 4$?
6. (10 points) Compute the equilibrium point \((E, F)\) of the dynamical system
\[
\begin{align*}
u_n &= 3u_{n-1} - 2v_{n-1} - 4 \\
v_n &= 5u_{n-1} - 3v_{n-1} - 28
\end{align*}
\]

7. (Bonus 10 points) a. Describe the long term behavior of the solution to the dynamical system \(w_n = -0.2w_{n-1} + 24, \ w_0 = 40\); that is, what happens to \(w_n\) as \(n \to \infty\)? Does \(w_n\) increase, decrease, oscillate, tend towards or away from the equilibrium?

b. What is the equilibrium value and is it stable or not? Explain.

c. Express \(w_{n+1}\) in terms of \(w_n\).