MATH 172 Spring, 2002 Exam #1 Name:______________________

There are 100 points. For full credit you must show your work.

1. (6 points) Fill in the blank spaces in the folowing table.

n	a_n	Δa_n	$\Delta^2 a_n$
0	-10		3
1	-9		3
2			3
3			3

2. (10 points) In constructing a table of second differences of a sequence $\{a_n\}$ you find that they are all the same number, 3. Also you know that $a_0 = 1$. Which of the following is most likely to be the correct formula for a_n, and why? Show calculations, or argue by analogy, but give some reason for picking one over the rest. (a) $a_n = n^3 + 1$, (b) $a_n = 3n^2 - n + 1$, (c) $a_n = (3/2)n^2 - (1/2)n + 1$.

3. (14 points) Formulate a model in which the change in the amount of drug in the bloodstream Δa_n from one day to the next is proportional by a factor of -0.6 to a_n, the amount at the beginning of the day, plus a maintenance dose of

60 mg/day. Rewrite this equation, giving a_{n+1} in terms of a_n and constants. Compute the steady state amount of the drug in the bloodstream.

4. (25 points) Solve the difference equation $b_{n+1} = -0.2b_n + 24$, $b_0 = 25$. Describe the long term behavior of this solution; that is, what happens to b_n as $n \to \infty$. Does b_n increase, decrease, oscillate, tend towards or away from the equilibrium?

5. (25 points) Data sugget that a population follows the logistic model $P'(t) = (1/2)P(t)(1 - (P(t)/80))$.

a. Give a formula for the per capita growth rate in this model.

b. Does this model have any steady state(s) or equilibrium value(s)? Show graphically what happens over the long term if $P_0 = 20$ and if $P_0 = 100$. Discuss the stability/instability of the steady state(s).

c. If $P(t)$ is very close to 0 then $P'(t)$ can be approximated by a simpler equation. Write this equation, and solve it, assuming $P_0 = 2$. What kind of growth is exhibited in this case?

6. (20 points) Compute the sum of each series; or state that no sum exists and explain why not.

a. $\sum_{k=1}^{\infty}(4/3)(3/5)^k$

b. $\sum_{k=1}^{\infty}(3/4)(5/3)^k$

c. $\sum_{k=0}^{\infty}(7/4)(-3/4)^k$