MATH 172X Spring, 2001 Exam #1 Name:____________________

There are 100 points. For full credit you must show your work. If you use your calculator for anything more than simple arithmetic, say so!

1. (10 points) In constructing a table of second differences of a sequence a_n you find that they are all the same number, 4. Also you know that $a_0 = 1$. Which of the following is most likely to be the correct solution to the problem, and why? (Show calculations, or argue by analogy, but give some reason for picking one over the rest.) (a) $a_n = n^4 + 1$, (b) $a_n = 3n^2 - n - 1$, (c) $a_n = 2n^2 + 1$.

2. (20 points) Formulate a model in which the change in the amount of drug in the bloodstream Δa_n from one day to the next is proportional by a factor of -0.4 to a_n, the amount at the beginning of the day, plus a maintenance dose of 2 mg/day. Then compute the long term steady state ($\Delta a_n = 0$) value of the drug in the bloodstream.

3. (20 points) A colleague suggests that a population follows a logistic model $a_{n+1} = a_n + 2a_n(1 - (a_n/40))$. You propose testing it by starting with $a_0 = 20$, and computing a_1 and a_2 . At this point you say that you didn't really know the initial value, and you try it again starting with $a_0 = 30$. Now you make a prediction about the model, and are ready to go out into the field and test it. What is this prediction? (Suggestion: does the model have any steady states (equilibrium values) and do they appear to be stable?)

4. (25 points) Let vectors $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and matrix $A = \begin{bmatrix} 1.3 & -0.2 \\ 0.15 & 0.9 \end{bmatrix}$.

a. Compute $3\mathbf{v} + \mathbf{w}$.

Just to recap, $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and matrix $A = \begin{bmatrix} 1.3 & -0.2 \\ 0.15 & 0.9 \end{bmatrix}$.

b. Plot $\mathbf{v}$ and $A\mathbf{v}$.

c. The eigenvalues for this matrix are $\lambda = 1.2$ and $\lambda' = 1$, and eigenvectors are $\mathbf{v}$ and $\mathbf{w}$. Which goes with which? Explain.

d. One of the given vectors represents a steady state. Find a vector that lines up with this one, but whose column total is 1.

5. (25 points) We have a predator-prey (fox-rabbit) model in which

$$\begin{aligned} R_{n+1} &= 1.3R_n - 0.2F_n \\ F_{n+1} &= 0.15R_n + 0.9F_n \end{aligned}$$

The matrix of coefficients is the same $A = \begin{bmatrix} 1.3 & -0.2 \\ 0.15 & 0.9 \end{bmatrix}$ as above, so we also have the eigenvectors $\mathbf{v}$, $\mathbf{w}$, and eigenvalues λ, λ'.

a. Give a formula for the fox population F_k in terms of k and the initial population F_0 if there are no rabbits. What happens to the fox population F_k in the long run?

b. We know that this model has a general solution $\begin{bmatrix} R_k \\ F_k \end{bmatrix} = A^k \begin{bmatrix} R_0 \\ F_0 \end{bmatrix}$. Using an initial population vector $\begin{bmatrix} R_0 \\ F_0 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix} = 4\mathbf{v} + \mathbf{w}$, and the eigenvalue-eigenvector relationships that you found earlier, describe the long term behavior of $\begin{bmatrix} R_k \\ F_k \end{bmatrix}$. Which part of the answer is dominant and why?