**MATH 122** **Exam #3** **Name:**______________________
**Fall, 2000**

There are 100 points. For full credit you must show your work.

1. (32 points) Compute the derivative. Your answers should have the form "derivative (in suitable notation) equals formula answer".

   a. $P = x^8 - \frac{4}{5}\sqrt{x} + \frac{4}{x^3}$

   b. $y = te^{-0.5t}$

   c. $z = \ln(r^4 + 1)$

   d. $f(w) = (5w^3 - 2 + e^{4w})^6$

2. (18 points) a. Compute $\displaystyle\int_1^5 6\ln(x/2)\,dx$ **by hand**, using 4 equal divisions of length $\Delta x =$ ______ . Show the table of values and the sums! Give the right sum, the left sum, and their average; give all numbers to 2 decimal places.

b. Give the built-in calculator value for this integral. ______ (If you wish to plot the function, I recommend a window with $1 \le x \le 5$ and $-4 \le y \le 6$ .)

3. (15 points) The graph of $y = f'(x)$ is given below. (This is the derivative of another function!)

a. Compute $\int_0^6 f'(x)\,dx$ exactly. ______

b. If we also know that $f(0) = 1$ , then $f(6) =$ ______ .

c. For which value(s) of $x \neq 0$ does the graph of the function $f(x)$ itself have critical point(s), and do these represent a local min, local max, or neither for $f$ ? Explain your answer briefly.

4. (5 points) Consider a function $h(x)$ whose only known values are $h(1) = 2$, $h(3) = 6$, $h'(1) = -2$, $h'(3) = 4$. Calculate $\int_1^3 h'(x)\,dx =$ ______ .

5. (20 points) The graph of $C'(q)$, that is, of marginal cost, is shown below. Production units are in kilograms (Kg); and $C'$ is measured in \$/Kg. What does the quantity $\int_{10}^{40} C'(q)\,dq$ represent in real world terms? Also give the units in which it is measured, and its approximate numerical value. Finally, show how to represent it on the graph.

6. (10 points) Consider the integral $\int_1^4 g(t)\,dt$ for a certain **monotonic** function $g$. This means that $g$ is either increasing or decreasing on the entire interval $1 \le t \le 4$; the graph does not turn around.

a. I have computed the left and right sums, and their averages, for the specified numbers of rectangles, rounded off to **3** decimal places.

| $n$ | left sum | right sum | average |
|---|---|---|---|
| 2 | $-1.312$ | 12.188 | 5.438 |
| 50 | 3.483 | 4.023 | 3.753 |
| 100 | 3.616 | 3.886 | 3.751 |
| 400 | 3.716 | 3.784 | 3.750 |
| 600 | 3.728 | 3.773 | 3.750 |

Based on this evidence, to 3 decimal places, $\int_1^4 g(t)\,dt =$ ______ .

b. The function $g$ is increasing / decreasing (circle one) for $1 \le t \le 4$. Briefly explain **using a diagram** how you know this.