MATH 122 Exam #3 Spring, 2004 Name:____________________

For full credit you must show your work, and give your reasoning, or other evidence for you answer.

1. (10 points) Consider the integral $\int_0^5 f(x)\,dx$ for a certain **monotonic** function f. This means that f is either increasing or decreasing on the entire interval $0 \le x \le 5$; the graph does not turn around.

   a. I have computed the left and right sums, and their averages, for the specified numbers of rectangles, rounded off to **3** decimal places.

n	left sum	right sum	average
4	6.417	10.417	8.417
16	7.572	8.572	8.072
64	7.924	8.174	8.049
256	8.016	8.079	8.047
1024	8.039	8.055	8.047

   The best estimate of the value of the integral, to 3 decimal places, based on the table above, is ______________ .

   b. The function f is increasing / decreasing (circle one) for $0 \le x \le 5$. Briefly explain (verbally and/or graphically) how you know this.

2. (15 points) The graph of $y = F'(x)$ is given below.

   a. The numerical value of $\int_0^8 F'(x)\,dx$ is ______________ .

   b. According to the Fundamental Theorem of Calculus $\int_0^8 F'(x)\,dx =$ ______________ .

   c. If $F(0) = -3$, then $F(8) =$ ______ .

3. (18 points) Compute the indefinite integral, that is, the **general antiderivative**.
$\int (3u^2 + \frac{1}{4}\sqrt{u} + 7 + \dfrac{5}{u^4})\, du$

4. (7 points) Compute $F(t) = \int e^{2t}\, dt$. For a 4 point bonus, if $F(0) = 3/2$, then $C =$ _______ .

5. (25 points) The graph of marginal cost $C'(q)$ is shown below, where production q is measured in tons, and $C'(q)$ is measured in units of \$100/ton.
   a. $C'(0) =$ _______________ . Use this value to estimate as well as possible the cost in dollars, above and beyond the fixed costs, that it takes to produce the first 0.2 tons. Is this an over- or under-estimate, and why?

b. What are the units of $\int_1^3 C'(q)\,dq$ and what does this quantity represent in real world terms? (Suggestion: what does the "area" of one box really represent?)

c. From the graph we obtain these values

q	1	2	3
$C'(q)$	3.4	4.3	5.5

Use these values to compute, as accurately as possible, an approximation to the value of $\int_1^3 C'(q)\,dq$.

6. (25 points) Let $g(x) = x - \ln(x)$. a. Compute $\int_{0.1}^{1.6} g(x)\,dx$ any way you like to **3** decimal places. Accuracy matters, so if you want partial credit for a wrong answer, be sure to indicate how you got your answer.

b. Compute $g'(x)$.

c. Find the exact value(s) of x at which $g(x)$ has its global maximum for $0.1 \le x \le 1.6$ and find the exact value(s) of x at which $g(x)$ has a local minimum. You may use your calculator to check your answer, but you must use derivative methods to find and justify your answer.

7. (10 bonus points) Use the graphs shown below to answer the questions.
   a. In the graph on the right clearly indicate which is the graph of f' and which is the graph of f''.

   b. f has its steepest positive slope at $x =$ ______________ . At this x-value f'' is (circle one) positive / zero / negative. Give your answer correct to one decimal place; for this you cannot use the graph of $f(x)$ alone, but must make use of the other graphs as well.