MATH 122   Spring, 2004   Exam #2   Name:_____

There are 100 points. **For full credit you must show your work**. Calculators are allowed, but may NOT be shared.

1. (20 points) It costs $C(q)$ dollars to produce q ounces of perfume.
   a.   Interpret $C(200) = 18,000$ and $C'(200) = 25$ _____ (fill in the units) in real world terms.

   b.   Give the **best possible** estimate for the cost of producing 203 ounces.

   c.   What is the average cost per ounce at a level of production of 200 ounces? Is this greater than or less than the marginal cost per ounce, and how do you explain the discrepancy?

   d.   The revenue function for this product is $R(q) = 45q$ dollars for q ounces. Does it pay to increase production above 200 ounces? Explain in terms of marginal revenue and marginal cost.

2. (5 points) On many islands the number of different species N is proportional to the cube root of the area A. If an island is observed to have an area of 1000 Km^2 and 320 species, compute the constant of proportionality k.

3. (15 points) Let $y = h(x) = 5(0.6)^x$. Graph this function for $0 \le x \le 6$.

    a.   Compute the slope of this graph at $x = 3$ (and say/show how you did it, which might be by doing part (b) first). Write a formula for the tangent line at $x = 3$, and show the tangent line on your graph.

    b.   Zoom in on the graph to estimate the instantaneous rate of change of h at at $x = 3$. **Keep at least 5 decimal place accuracy in your work, but give your answers correctly rounded to 3 decimal places.** "To compute the slope, I used the points (_____ , _____) and (_____ , _____). I chose these points because this part of the graph _____ _____ , and these points are _____ _____ _____ ." Illustrate the portion of the graph and these points on it in the window given below. "I conclude that the instantaneous rate of change of $y = h(x)$ at $x = 3$ is _____ $\approx$ _____ " (Give your answer in a correct notation.)

4. (30 points) Compute the formula for the derivative. Your answer should be in the form of an equation in which the left hand side gives the derivative in correct mathematical notation, and the right hand side gives the actual formula.

   a. $w = 6x^4 - \sqrt{x^2 + 7}$

   b. $g(r) = \dfrac{r}{4} + 5 - \dfrac{4}{r}$

   c. $P(t) = 100e^{-.02t}$

   d. $L = \ln(5z^4 + 10)$

5. (10 points) Given the graph of $y = h(x)$ shown below, sketch the graph of $y = h'(x)$. You may want to make a "table of values" with at least verbal descriptions of the derivative at various values of x.

6. (13 points) Consider the cost and revenue functions shown below for a company that makes DVD's. The production q is measured in thousands of DVD's, and the money is measured in tens of thousands of $'s.

a. Use the graph to compute the marginal revenue, assuming that the revenue $R(q)$ is a linear function. Don't forget to give the units!

b. At what production levels does the company actually make a profit? _____ $\leq q \leq$ _____

c. The production level that generates the maximum profit is $q =$ _____ . Explain how you found this value.

7. (7 points) Given the following data, estimate $g'(0.4)$ as well as possible.

t	0	0.2	0.4	0.6	0.8	1.0
$g(t)$	7.9	6.3	5.8	4.9	3.1	3.9