# 1. (a) Using the standard unit vector method to find M:

- M leaves the y-axis fixed, so \( M e_2 = e_2 \).
- M rotates the z-axis to the x-axis: \( M e_3 = e_1 \).
- M rotates the x-axis to the new z-axis: \( M e_1 = -e_3 \).

Thus: \[
M = \begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
-1 & 0 & 0
\end{bmatrix}
\]

(b) Using the standard unit vector method to find M:

- M leaves the x-axis fixed: \( M e_1 = e_1 \).
- M maps the +z-axis to the -z-axis: \( M e_3 = -e_3 \).
- M maps the +y-axis to the -y-axis: \( M e_2 = -e_2 \).

Thus: \[
M = \begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{bmatrix}
\]

(c) Using the standard unit vector method to find M:

- M leaves the x- and z-axes unchanged: \( M e_1 = e_1 \).
- M leaves the x- and z-axes unchanged: \( M e_3 = e_3 \).

- M maps the +y-axis to the -y-axis: \( M e_2 = -e_2 \).

Thus: \[
M = \begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

# 3. (a) \[
A = \begin{bmatrix}
2 & 1 & -7 \\
5 & 3 & -1
\end{bmatrix}
\]

Note that \( A \) is in echelon form.

To solve \( Ax = 0 \) we have:

\[
2x_1 + x_2 - 3x_3 = 0 \\
3x_2 - x_3 = 0
\]

Let \( x_3 \) be the free variable.
Then \( x_2 = \frac{1}{3} x_3 \)
and \( x_1 = \frac{1}{2} (3 x_3 - x_2) = \frac{1}{2} \left( \frac{8}{3} x_3 \right) = \frac{4}{3} x_3. \)

The general solution to \( Ax = 0 \) is
\[ x = x_3 \begin{bmatrix} \frac{4}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix} \]

The kernel of \( T(x) = Ax \) is \( \text{Span} \left\{ \begin{bmatrix} \frac{4}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix} \right\} = \text{Span} \left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \right\} \).

(6). If \( T(x) = Bx : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) (i.e. \( B \) is a \( 2 \times 3 \) matrix).

Then there must be at least one free variable when we reduce the augmented matrix \([B | 0]\) to echelon form.

Because there is a free variable, there is a non-trivial solution to \( Bx = 0 \) so the kernel of \( T \) contains a nonzero vector.

Note: We could also state that the columns of \( B \) are linearly dependent (because any set of \( k \) vectors in \( \mathbb{R}^n \) with \( k > n \) must be linearly dependent).

and so there is a linear dependence relation between the columns, i.e. there is a nonzero solution to \( Bx = 0 \).