Math 511
Meade
H.W. Solutions
1/23/04

6. \( P(A) = 0.4 \), \( P(B) = 0.5 \), \( P(\overline{A} \cap \overline{B}) = 0.3 \)
   a. \( P(\overline{A} \cup \overline{B}) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.3 = 0.6 \)
   b. \( P(A \cup \overline{B}) = P(A) - P(\overline{A} \cap B) = 0.4 - 0.3 = 0.1 \)
   c. \( P(\overline{A} \cup \overline{B}) = P(\overline{A} \cap \overline{B}) = 1 - P(A \cap B) = 1 - 0.3 = 0.7 \) (DeMorgan's Law)

9. \( P(A_i) = \frac{1}{3}, \) \( i = 1, 2, 3 \): \( P(A_1 \cap A_2 \cap A_3) = \left(\frac{1}{3}\right)^2 \), \( i \neq j \):
   a. \( P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) \)
     \[ = 3 \left(\frac{1}{3}\right) - 3 \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 = (27 - 9 + 1) / 27 = \frac{19}{27} \]
   b. \( 3 \left(\frac{1}{3}\right) - 3 \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 = 1 - \frac{9}{27} + \frac{1}{27} = 1 - \frac{8}{27} = 1 - \left(\frac{2}{3}\right)^3 = 1 - (1 - \frac{1}{3})^3 \)

10. Prove Thm. 2.1-6
    \[ A \cup B \cup C = A \cup (B \cup C) \text{, by associativity of set operations} \]
    \[ P(A \cup (B \cup C)) = P(A) + P((B \cup C)) - P(A \cap (B \cup C)) \] (by Thm. 2.1-5)
    \[ = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap (B \cup C)) \] (by Thm. 2.1-5)
    \[ = P(A) + P(B) + P(C) - P(B \cap C) - P((A \cap B) \cup (A \cap C)) \] (by distribution)
    \[ = P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)] \]
    \[ = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B \cup (A \cap C)) \]
    \[ = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \]
    Since \( P(A \cap B) = P((A \cap B) \cap C) \), by commutativity and associativity, and \( P(A \cap B) = P(A \cap B) \), by
    the idempotent law of set theory.
11. Let \( n \in \mathbb{N} \), and \( P(\epsilon n) = (\frac{1}{2})^n \).

a. \( A = \{ n : 1 \leq n \leq 10 \} \); find \( P(A) = P(1) U P(2) U \ldots U P(10) \)
\[
P(A) = \sum_{i=1}^{10} \left( \frac{1}{2} \right)^i = \sum_{i=0}^{10} \left( \frac{1}{2} \right)^i - 1 = \frac{1 - \left( \frac{1}{2} \right)^{11}}{1 - \frac{1}{2}} = 2 - \left( \frac{1}{2} \right)^{10} - 1 = 1 - \frac{1}{1024} = \frac{1023}{1024}
\]

b. \( B = \{ n : 1 \leq n \leq 20 \} \), find \( P(B) \)
\[
P(B) = 1 - \left( \frac{1}{2} \right)^{20} = \frac{1}{1,048,575}
\]

By the reasons as in A.

c. \( P(A U B) \); since \( A \subset B \), the \( P(A U B) = P(\text{the greater set}) = P(B) \)

d. \( P(A \cap B) \); since \( A \subset B \), the \( P(A \cap B) = P(\text{the lesser set}) = P(A) \)

e. \( C = \{ n : 11 \leq n \leq 20 \} \), find \( P(C) \).

Notice that: \( B = A U C \), so \( P(B) = P(A) + P(C) \), by the 3rd axiom of probability.

so \( P(C) = P(B) - P(A) = \frac{1023}{1,048,576} \)

f. Find \( P(B') = 1 - P(B) = \frac{1}{2^{20}} \)

12. Let \( x \in [0,1] \), if \( x \) is randomly selected, then guess by intuition the following.

a. \( \emptyset \ P(\epsilon x : 0 \leq x \leq y_{33}) = \frac{1}{3} \)

b. \( P(\epsilon x : y_{33} \leq x \leq 13) = \frac{2}{3} \)

c. \( P(\epsilon x : x = y_{33}) = 0 \)

d. \( P(\epsilon x : y_{2} \leq x \leq 53) = P(\epsilon x : y_{2} \leq x \leq 13) = \frac{y_{2}}{2} \)
13. Let our set be a roulette table which has 2 green slots, 18 black slots, and 18 red slots.

a. What is the sample space, $S$?
   $S = \{0, 00, 1, 2, \ldots, 36\}$

b. Let $A = \{0, 00\}$; $P(A) = \frac{2}{38} = \frac{1}{19}$

c. Let $B = \{14, 15, 17, 18\}$; $P(B) = \frac{4}{38} = \frac{2}{19}$

d. Let $D = \{\text{odd}\}$; $P(D) = \frac{19}{38} = \frac{9}{19}$

15. 

\[ \overline{AB} \]

C divides $\overline{AB}$ into $\overline{AC}$ & $\overline{CB}$ such that $\overline{CB} = 2 \overline{AC}$

D does this in an analogous way.

So any point, $P$, between $A$ & $C$ or between $D$ & $B$ divides $\overline{AB}$ such that $2\overline{AP} \leq \overline{PB}$ or such that $2\overline{PB} \leq \overline{AP}$,

So $\frac{P(\overline{AC} \cup \overline{BD})}{3} + \frac{1}{3} = \frac{2}{3}$

So $P(\overline{P \in \overline{AC} \text{ or } P \in \overline{BD}}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$