Instructions:

1. There are a total of 9 problems on 10 pages. Check that your copy of the exam has all of the problems.

2. You must show all of your work to receive credit for a correct answer.

3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

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Have A Great Summer!
1. (9 points) Let $\mathbf{a} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - \mathbf{k}$, and $\mathbf{c} = (1, 6, 0)$. Find each of the following:

(a) $\mathbf{a} \cdot \mathbf{c}$

(b) $\mathbf{b} \times \mathbf{c}$

(c) $\mathbf{c} \cdot \mathbf{c} - |\mathbf{c}|^2$

2. (9 points)

(a) What is the direction of the line $x = 2 - 3t$, $y = 3t$, $z = 2 - t$? What is a point on the line? NOTE: Be sure to clearly label the two answers.

(b) Find parametric equations for the line through $(6, 1, -3)$ and $(-2, 1, 3)$.

(c) Find an equation for the plane containing the lines $x = 3t$, $y = 1 + t$, $z = 4 + t$ and $x = 3 - 2s$, $y = 2$, $z = 5 + 4s$. 
3. (10 points) Let $C$ be the parametric curve $x = t^2$, $y = t^3$, $z = 3$.

(a) Find all points on the curve with $x = 4$.

(b) Find the tangent line to the curve at the point $(1, -1, 3)$.

(c) Find the equation of the normal plane to the curve at the point $(1, -1, 3)$.

(d) Find the speed of a particle that follows this curve as a function of $t$.

(e) Find all points where the particle’s speed is zero.
4. (12 points) Consider the curve \( \mathbf{r}(t) = (t \cos t, t \sin t, 2t) \). Find each of the following:

(a) \( \mathbf{r}'(\frac{\pi}{2}) \)

(b) \( \mathbf{T}(\frac{\pi}{2}) \)

(c) \( \mathbf{r}''(\frac{\pi}{2}) \)

(d) a definite integral for the length of the curve between the points (0, 0, 0) and \((-\pi, 0, 2\pi)\).

**Note:** Do not evaluate this integral.
5. (15 points) Let \( f(x, y) = 3x^2y^4 + 7\frac{x^2}{y^3} \). Find

(a) \( \nabla f(2, 1) \)

(b) \( f_{xx} \)

(c) \( \frac{\partial^2 f}{\partial x \partial y} \)

(d) the directional derivative of \( f \) at (2, 1) in the direction of \( \mathbf{a} = \mathbf{i} - \mathbf{j} \)

(e) \( \frac{\partial f}{\partial t} \) where \( x = e^t \) and \( y = e^{-t} \)

**Note:** Your answer should be expressed in terms of \( t \).
6. (9 points) Find each limit or explain why it does not exist.

(a) \[ \lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^4 - y^4} \]

(b) \[ \lim_{(x,y) \to (0,0)} \frac{x^4 - y^4}{x^2 + y^2} \]

(c) \[ \lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + y^2} \]
7. (12 points) Let \( f(x, y) = x^2 y - 6x^2 - 3y^2 \).

(a) Find all critical points of \( f \).

(b) Classify each of the critical points as a local maximum, local minimum, or saddle point, if possible.
8. (12 points)

(a) Evaluate \( \int_{1}^{2} \int_{3}^{x} \int_{0}^{\sqrt{3y}} \frac{z}{y^2 + z^2} \, dz \, dy \, dx \).

(b) Sketch the domain of integration for the iterated integral \( \int_{0}^{1} \int_{y}^{\sqrt{y}} f(x, y) \, dx \, dy \).

(c) Interchange the order of integration in the integral in (b).
9. (12 points) Let $S$ be the solid bounded below by $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 4$. The iterated triple integral for the volume of $S$ is

$$V = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{-y}^{\sqrt{4-x^2-y^2}} dz \, dy \, dx.$$ 

(a) Show that the top and bottom surfaces intersect in the circle with radius $\sqrt{2}$ centered at the origin.

(b) Convert the triple integral for the volume of $S$ to cylindrical coordinates.

(c) Convert the triple integral for the volume of $S$ to spherical coordinates.
(d) Evaluate one of the three triple iterated integrals for the volume to show that the volume of $S$ is $V = \frac{8}{3}(2 - \sqrt{2})\pi$. 