Exam 1
February 20, 2001

Name: ______________________
SS #: ______________________

Instructions:

1. There are a total of 7 problems on 8 pages. Check that your copy of the exam has all of the problems.

2. You must show all of your work to receive credit for a correct answer.

3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

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Good Luck!
1. (15 points) Let \( \mathbf{a} = 2\mathbf{i} - 5\mathbf{j} - \mathbf{k} \), \( \mathbf{b} = 3\mathbf{i} - \mathbf{j} \), and \( \mathbf{c} = \langle 1, 0, -6 \rangle \). Find each of the following:

(a) \( \mathbf{a} \cdot \mathbf{c} \)

(b) \( \mathbf{b} \times \mathbf{c} \)

(c) \( \mathbf{c} \cdot \mathbf{c} - |\mathbf{c}| \)
2. (15 points) Let $C$ be the parametric curve $x = t, y = t^2, z = 3$.

(a) Find all points on the curve with $y = 4$.

(b) Find the tangent line to the curve at the point $(-1, 1, 3)$.

(c) Find the equation of the normal plane to the curve at the point $(-1, 1, 3)$.

(d) Find the speed of a particle that follows this curve.

(e) Find all points where the particle’s speed is zero. Explain your answer.
3. (15 points)

(a) What is the direction of the line \( x = -3 + 2t, \ y = 3, \ z = -1 + 2t? \)

(b) Find parametric equations for the line through \((6, 1, -3)\) and \((-2, 1, 3)\).

(c) Find the center and radius of the sphere with equation \( x^2 + y^2 + z^2 + 6x - 2y + 5 = 0 \).
4. (20 points) Consider the curve $\mathbf{r}(t) = e^t \sin t \mathbf{i} + e^t \cos t \mathbf{j} + e^t \mathbf{k}$, $1 \leq t \leq 5$. Find each of the following:

(a) $\mathbf{r}'(\pi)$

(b) $\mathbf{T}(\pi)$

(c) $\mathbf{r}''(\pi)$
5. (8 points) Find all unit vectors that are perpendicular to both of the vectors $a = 3i - 3j + k$ and $b = -i - 2j + 4k$. 
6. (15 points) Sketch the level curves of $f(x, y) = x^2 + y^2$ for $k = -1, 0, 1, 4, 9$. 
   *Remember to label the curves and the axes.*
7. (12 points) Let \( f(x, y) = 3x^2y^4 + 7 \frac{x^2}{y^3} \). Find

(a) \( \frac{\partial f}{\partial x} \)

(b) \( \frac{\partial^2 f}{\partial x^2} \)

(c) \( f_{xy} \)