Math 241 (Section 502)
Prof. Meade

Exam 1
February 20, 2002

University of South Carolina
Spring 2002

Name: $\qquad$
SS \#: $\qquad$

Instructions:

1. There are a total of ?? problems on ?? pages. Check that your copy of the exam has all of the problems.
2. You must show all of your work to receive credit for a correct answer.
3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 24 |  |
| 2 | 28 |  |
| 3 | 24 |  |
| 4 | 12 |  |
| 5 | 12 |  |
| Total | 100 |  |

## Good Luck!

1. (24 points) Let $\mathbf{a}=3 \mathbf{i}-5 \mathbf{j}-2 \mathbf{k}, \mathbf{b}=3 \mathbf{i}-\mathbf{j}$, and $\mathbf{c}=\langle 1,0,-6\rangle$. Find each of the following that is defined. If a quantity is not defined, explain why.
(a) $\mathbf{a} \cdot \mathbf{c}$
(b) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$
(c) $\mathbf{b} \times \mathbf{c}$
(d) $\mathbf{c} \cdot \mathbf{c}-|\mathbf{c}|$
(e) $\mathbf{a} / \mathbf{b}$
2. (28 points) A fly is crawling along a wire helix so that its position is $\mathbf{r}(t)=\langle 6 \cos (\pi t), 6 \sin (\pi t), 2 t\rangle$ for $t \geq 0$. The wire is inside a spherical balloon with radius 10 centered at the origin.
(a) What is the Cartesian equation for the spherical balloon?
(b) At what time does the fly reach the sphere?
(Hint: the fly hits the sphere at the point $P(6,0,8)$.)
(c) Write the equation of the tangent line to the fly's motion at $t=4$.
(d) What is the curvature of the fly's motion at $t=4$ ?
3. (24 points) A plane through the point $P(2,-4,-5)$ is perpendicular to the line joining the points $Q(-1,5,-7)$ and $R(4,1,1)$.
(a) Find parametric equations for the line through $P$ and $Q$.
(b) Find a Cartesian equation of the plane.
(c) Sketch the plane by drawing its traces.

Note: Be sure to label the axes in the sketch.
4. (12 points)


Match the plot with the appropriate class of quadric surfaces.
(a) $\qquad$ Hyperboloid of one sheet
(b) $\qquad$ Hyperboloid of two sheets
(c) $\qquad$ Ellipsoid
(d) $\qquad$ Hyperbolic Paraboloid
(e) $\qquad$ Elliptic Cone
(f) $\qquad$ Paraboloid
5. (12 points)
(a) What are the Cartesian coordinates of the point $P$ with cylindrical coordinates $(8, \pi / 3,2 \pi / 3)$ ?
(b) What are the Cartesian coordinates of the point $Q$ with spherical coordinates ( $8, \pi / 3,2 \pi / 3)$ ?
(c) Write the equation $x^{2}+y^{2}=z^{2}$ in spherical coordinates.
(d) Write the equation $x^{2}+y^{2}=z^{2}$ in cylindrical coordinates.

