Instructions:

1. There are a total of 4 problems

2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and do not involve unreasonable manual calculations.

3. Be sure you answer the questions that are asked.

4. Copy your final answer to each question to the back of this page.

5. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.

6. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure to put your name on each sheet, include the question number for all work, and staple all pages — in order — to this page when you turn in your completed test.

7. Check your work. If I see clear evidence that you checked your answer (when possible) and you clearly indicate that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
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Happy Thanksgiving!
1. (12 points) Values for a function \( f(x, y) \) defined on \( R = [1, 5] \times [2, 6] \) are given in the table below.

<table>
<thead>
<tr>
<th>( y \times x )</th>
<th>1</th>
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Estimate \( \iint_R f(x, y) \, dA \) with a Riemann sum with \( m = 2, \ n = 2 \), and take the sample points to be the upper-right corner of each square.

\[
\iint_R f(x, y) \, dA \approx (f(3,4) + f(5,4) + f(3,6) + f(5,6)) \, \Delta A = (0 + 8 + 0 + 0) \cdot 4 = -8
\]

2. (12 points) Calculate the iterated integral \( \int_0^1 \int_{-1}^3 y e^{xy} \, dy \, dx \).

\[
\int_0^3 \int_{-1}^1 y e^{xy} \, dy \, dx = \int_0^1 e^x \left[ \int_{-1}^3 y e^{xy} \, dy \right] dx = \int_0^1 e^x \left[ -\frac{1}{e^x} \right]_0^3 dx = \int_0^1 e^x \left( e^3 - 1 \right) dx = \left[ e^3 - e^x \right]_0^1 = e^3 - e - 4
\]

3. (51 points) Evaluate each definite integral.

(a) \( \int \int_T \frac{1}{1 + x^2} \, dA \) where \( T \) is the triangular region with vertices \( (0, 0), (1, 0), (1, 1) \).

\[
\int_0^1 \int_0^{x^2} \frac{1}{1 + x^2} \, dy \, dx = \int_0^1 \left[ \arctan(y) \right]_0^{x^2} dx = \int_0^1 \frac{1}{2} \arctan(2x) - \frac{1}{2} \arctan(x) \, dx
\]

(b) \( \int \int_D (x^2 + y^2) \, dA \) where \( D \) is the region in the first quadrant bounded by the lines \( y = 0 \) and \( y = \sqrt{3}x \) and the circle \( x^2 + y^2 = 9 \).

\[
\int_0^{\pi/3} \int_0^{\sqrt{3} \sin \theta} \frac{1}{2} \rho^2 \, d\rho \, d\theta = \frac{1}{2} \int_0^{\pi/3} \left[ \frac{\rho^3}{3} \right]_0^{\sqrt{3} \sin \theta} d\theta = \frac{1}{2} \int_0^{\pi/3} \frac{3 \sin^3 \theta}{3} d\theta = \frac{1}{2} \theta \bigg|_0^{\pi/3} = \frac{\pi}{4}
\]

(c) \( \int \int_E \sqrt{z} \, dV \) where \( E \) lies above the plane \( z = 0 \), below the plane \( z = y \), and inside the cylinder \( x^2 + y^2 = 9 \).

\[
\int_0^{2\pi} \int_0^3 \int_0^{\sqrt{9 - r^2}} \sqrt{z} \, dz \, dr \, d\theta = \frac{2}{3} \left[ 9 \sqrt{9 - r^2} - \frac{3}{8} \theta \right]_0^{2\pi} = \frac{2}{3} (9 \pi)
\]

4. (25 points) Let \( S \) be the solid under the paraboloid \( z = x^2 + 4y^2 \) and above the rectangle \( R = [0, 2] \times [1, 4] \).

(a) Express the volume of \( S \) as an iterated (triple) integral.

\[
V = \iiint_S dV = \int_0^2 \int_1^4 \int_0^{x^2 + 4y^2} \, dz \, dy \, dx
\]

(b) Evaluate the integral found in (a).

\[
V = \int_0^2 \left[ \int_1^4 (x^2 + 4y^2) \, dy \right] dx = \int_0^2 \left[ x^2 y + \frac{4}{3} y^3 \right]_1^4 dx = \int_0^2 \left( 4x^2 + \frac{64}{3} - \left( x^2 + \frac{4}{3} \right) \right) dx
\]

\[
= \int_0^2 \frac{3}{3} x^2 + 184 \, dx = \left[ \frac{3}{5} x^3 + 184 x \right]_1^2 = 8 + 168 = 176
\]