MATH 241 (Section 502)
Prof. Meade

Exam 2
29 October 2010

University of South Carolina
Fall 2010

Name: Key
SS # (last 4 digits): ____________

Instructions:

1. There are a total of 7 problems (not counting the Extra Credit problem).
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. Copy your final answer to each question to the back of this page.
5. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
6. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure to put your name on each sheet, include the question number for all work, and staple all pages — in order — to this page when you turn in your completed test.
7. Check your work. If I see *clear evidence* that you checked your answer (when possible) and you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	10	
6	16	
7	26	
Extra Credit	10	
Total	100	

Happy Halloween!

1. (12 points) Find each limit, or explain why it does not exist.

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{2xy}{x^2+2y^2}$

   along $y=0$: $\frac{2xy}{x^2+2y^2} = 0 \to 0$ as $x \to 0$

   along $y=x$: $\frac{2xy}{x^2+2y^2} = \frac{2}{3} \to \frac{2}{3}$ as $x\to 0$ $\Big\} \Rightarrow$ limit DNE

   (b) $\lim_{(x,y)\to(2,1)} \dfrac{2xy}{x^2+2y^2} = \dfrac{2(2)1}{2^2+2(1)^2} = \dfrac{4}{6} = \dfrac{2}{3}.$

2. (12 points) Find the equation of the tangent plane to $z = 3x^2 - y^2 + 2x$ at $(1, -2, 1)$.

   $\vec{n} = \langle f_x(1,-2), f_y(1,-2), -1\rangle = \langle 8, 4, -1\rangle$ so $8(x-1)+4(y+2)-(z-1)=0.$

3. (12 points) Find the points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

   $f(x)=$ $\nabla f = \langle 2x, 8y, -2z\rangle$, $\vec{n} = \langle 2,2,1\rangle$ $\Big\}$ parallel when $2x=8y,\ 2x=-4z \Rightarrow$ $(y=\frac{x}{4})$ $(z=-\frac{x}{2})$

   $\Rightarrow$ on surface when $4 = x^2+4(\frac{x}{4})^2-(-\frac{x}{2})^2 = x^2 \Rightarrow x=\pm 2, y=\pm\frac{1}{2}, z=\mp 1$

4. (12 points) Suppose $z = f(x, y)$ where $x = g(s, t)$, and $y = h(t)$. In addition,

$$\begin{array}{llll} g(1,2)=3, & g_s(1,2)=-1, & g_t(1,2)=4, & \\ h(1)=4, & h(2)=6, & h'(1)=-5, & h'(2)=10, \\ f_x(3,6)=7, & f_x(3,4)=-2, & f_y(3,6)=8, & f_y(3,4)=0. \end{array}$$

   - Find $\frac{\partial z}{\partial s}(1,2)$. $= f_x(3,6)\,g_s(1,2) = 7(-1) = -7$
   - Find $\frac{\partial z}{\partial t}(1,2)$. $= f_x(3,6)\,g_t(1,2) + f_y(3,6)\,h'(2) = 7(4)+8(10) = 108$

5. (12 points) Find the maximum rate of change of $f(x, y) = x^2y + \sqrt{y}$ at the point $(2, 1)$. In what direction does it occur? direction: $\nabla f(2,1) = \langle 4, 9/2\rangle$

   max rate: $|\nabla f(2,1)| = \frac{1}{2}\sqrt{145}.$

6. (14 points) The rectangular box without a lid with maximum volume is to be made from 12 m^2 of cardboard.

   (a) Formulate this as a constrained optimization problem that can be solved by Lagrange multipliers. max $V = xyz$

   s.t. $S = xy+2xz+2yz = 12$

   (b) Find, *but do not solve*, the system of equations that arises from the application of Lagrange multipliers to the problem found in (a). $\nabla V = \lambda\nabla S$: $yz = \lambda(y+2z)$

   $xz = \lambda(x+2z)$

   $xy = \lambda(2x+2y)$

   & $xy+2xz+2yz=12$

7. (26 points) Let $f(x, y) = 8xy^2 - x^2y^2 - xy^3$ and let D be the closed trianglar region in the plane with vertices $(0, 1)$, $(7, 1)$, and $(0, 8)$.

   HINT: $f(x, y) = xy^2(8 - x - y)$, $f(\frac{7}{2}, 1) = \frac{49}{4}$, $f(0, 4) = f(\frac{7}{2}, \frac{9}{2}) = 0$, and the solution to $2x + y = 8$, $2x + 3y = 16$ is $x = 2$, $y = 4$.

   (a) Why does the function f have a maximum value and a minimum value on D? (This answer will be one or two sentences, no formulas or equations.) Because f is a continuous function and D is a closed and bounded set in $\mathbb{R}^2$.

   (b) Find all critical points of f. (Do not attempt to classify the critical points.)

   $\nabla f = \langle y^2(8-2x-y), xy(16-2x-3y)\rangle = \vec{0}$ when 1) $y=0, x=8$, 2) $y=0$, $x=$anything 3) $x=0, y=8$, or 4) $x=2, y=4$

   (c) Find the maximum and minimum values of f on D. At what points are these extreme values obtained? 3) $y=8-x$, $0\le x\le 7$: $f(x,8-x)=0$. And $f(2,4)=64$ ← max

   3 edges: 1) $x=0$, $1\le y\le 8$: $f(0,y)=0$ ← min 2) $y=1$, $0\le x\le 7$: $f(x,1)=x(7-x)$: max @ $x=\frac{7}{2}$: $f(\frac{7}{2},1)=\frac{49}{4}$, min @ $x=0, x=7$: $f(\cdot,1)=0$.

Extra Credit (10 points) Let $z = xy + xe^{y/x}$. Show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z$.

$\frac{\partial z}{\partial x} = y + e^{y/x} + xe^{y/x}(-y/x^2)$

$\frac{\partial z}{\partial y} = x + xe^{y/x}(\frac{1}{x})$

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x(y + e^{y/x} - \frac{y}{x}e^{y/x}) + y(x + e^{y/x})$

$= 2xy + xe^{y/x}$

$= xy + z.$