

Example: # 22 (p. 284).

$\lim_{x \rightarrow 0^+} \tan x \ln x$ is an indeterminate form of type $0 \cdot \infty$.

We have two choices:

$$1) \tan x \ln x = \frac{\tan x}{(\ln x)^{-1}} \quad \left(\frac{0}{0} \right)$$

$$\text{or } 2) \tan x \ln x = \frac{\ln x}{(\tan x)^{-1}} \quad \left(\frac{\infty}{\infty} \right)$$

Either form can be used, but I believe it will be quicker to work with 2). Let's try it:

$$\begin{aligned} \lim_{x \rightarrow 0^+} \tan x \ln x &= \lim_{x \rightarrow 0^+} \frac{\ln x}{(\tan x)^{-1}} \\ &= \lim_{x \rightarrow 0^+} \frac{1/x}{-(\tan x)^{-2} \sec^2 x} \end{aligned}$$

$$= \lim_{x \rightarrow 0^+} - \frac{\tan^2 x}{x \sec^2 x}$$

$$= \lim_{x \rightarrow 0^+} - \frac{\sin^2 x}{x \frac{1}{\cos^2 x}}$$

$$= \lim_{x \rightarrow 0^+} - \frac{\sin^2 x}{x}$$

$$= \left(\lim_{x \rightarrow 0^+} -\sin x \right) \left(\lim_{x \rightarrow 0^+} \frac{\sin x}{x} \right)$$

$$= 0 \cdot 1$$

$$= 0.$$

This equality is valid only because the new limit does exist.

Note: This limit could be evaluated by l'H but it's not necessary.