Maplets for Calculus is a collection of 129 Maple applets for use by students and instructors in calculus and precalculus. Instructors employ the Maplets for Calculus in a variety of ways, including in-class demonstrations and lab exercises. Students find the Maplets for Calculus as an electronic study guide, even as a tutor without the tutor. In this brief document we list the mathematical topics addressed by the Maplets for Calculus. A few screenshots provide an indication of the general pedagogy, including the use of graphics. But, to get a full appreciation of the interaction provided by the Maplets for Calculus and how they can be used by instructors and students, give them a try for yourself!

Maplets for Calculus (M4C) is a collection of 129 Maple applets that are useful by calculus students and their instructors. The maplets in M4C provide interactive graphical user interfaces for many of the most common examples and exercises on a variety of topics in precalculus and calculus. While the M4C focused initially on single-variable topics, many of the 35 maplets new to M4C 1.3 address multivariate and vector-valued topics.

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Table 1: Development and distribution of content of the M4C. A full listing of maplets is provided at the end of this document.

The Maplets for Calculus balance the development of both understanding and technical skills. Most maplets allow for problems to be either algorithmically-generated or user-entered.

Students find the algorithmically-generated problems very effective when developing technical skills and for building confidence as they prepare for a quiz or exam. Immediate feedback and infinite patience are two features students find particularly helpful. Students have reported that the hints and step-by-step checking of their responses is as effective as a private tutor. For example, Figure 1 shows the Integration by Substitution maplet in which the user has omitted the differential in the substitution.

There are a multitude of opportunities for instructors to use the Maplets for Calculus to enhance their courses. The ability to launch a Maplet from a webpage, PowerPoint presentation, or PDF document makes it convenient to use a Maplet for demonstrations and examples in a lecture. The extensive use of graphics – 2D, 3D, and animation – add to the impact that the Maplets can have in the classroom. Examples include the Derivatives of Inverse Functions (Figure 2), Volume by Slicing (Figure 3) and Area as an Antiderivative (Figure 4).

Others prefer to develop a project or a lab that utilizes a Maplet in a guided-discovery exercise. For example, the Epsilon Delta Definition of a Limit maplet provides a graphical and interactive environment for exploring the $\varepsilon-\delta$ definition of a limit (Figure 5).

Instructors can also have their students drill on traditional topics such as Related Rates (Figure 6) and then apply these ideas to solve the classic Ladder Problem (Figure 7). Another robust collection of maplets focuses on optimization in general (Find and Classify Critical Points) and its classical applications (including separate maplets on traditional optimization applications such as minimizing the surface area of a can with a fixed volume, maximizing the enclosed area of a field divided into several pens, minimizing the surface area of a box with a fixed volume, minimizing the area of a rectangle inscribed in an ellipse, minimizing the distance from a point to a line, and minimizing the time needed to cross a river...)

Some of the 35 new maplets in M4C v. 1.3 extend the coverage of precalculus and single-variable calculus topics and introduces the first Maplets for Calculus for topics from calculus of multivariate and vector-valued functions.

Many of the precalculus maplets are used by students who need a refresher on specific topics. Two examples are the Solve Absolute Value Linear Equations (Figure 10, Pre-Calc: Alg & Geom # 6) and Finding Vertical Asymptotes (Figure 11, Pre-CalcC: Alg & Geom #11). Students can use the Show buttons to see a worked example, then work through enough examples to master the skill. Graphical representations of the problem, and each step in the solution process, help students develop a better conceptual understanding while simultaneously working to improve their mechanical skills.

The authors put a lot of effort into developing the collection of four maplets for iterative methods for solving an equation. The Bracket Method for Solving ... (Figure 12, Limits & Continuity: #32 & 34) and the Bisection Method for Solving ... (Limits & Continuity: #33 and 35) are very flexible, accepting any permissible guess at each step. They also have to keep track of the error, and use this to decide when to terminate the iterations.

The maplets for multivariate topics follow the same principles that have made the univariate maplets so successful. Some of the topics addressed include: Tangent Plane to a Level Surface (Figure 13, Partial Derivatives #121), Center of Mass of a Solid (Figure 14, Multiple Integrals #123 – beta), and Surface Area of Cones and Paraboloids (Figure 15, Surfaces and Surface Integrals. # 124).

After a listing of all 129 Maplets into 16 different categories, the remainder of this paper shows snapshots taken from several different maplets in the M4C collection.
Maplets for Calculus v. 1.3 – Table of Contents

* New in M4C 1.3
+ Free at MYMathApps.com, Try Before you Buy

Precalculus: Algebra & Geometry
1. Basic 7 Functions (2D)
2. Shifting Functions (2D)
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33. Bisection Method of Solving a Continuous Equation using Numeric Data (2D) *
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37. From Secant Slopes to Tangent Slope, using a Formula
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40. Derivative Drill +
41. Implicit Differentiation
42. Logarithmic Differentiation
43. Derivatives of Inverse Functions (2D) +

Derivatives: Applications
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46. Related Rates: The Ladder (2D animation)
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52. Properties of the Graph of a Function (2D)
53. Properties of the Graph of the First Derivative (2D)
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55. Finding & Classifying Critical Points *
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57. Max/Min: Fence a Field (2D) *
58. Max/Min: Surface Area of a Box (2D animation) *
59. Max/Min: Rectangle in Ellipse (2D animation) +
60. Max/Min: Distance from a Point to a Line (2D) *
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64. Anti-Derivatives with Initial Conditions *
65. From Acceleration to Velocity and Position
66. Area as an Anti-Derivative: Derive the Fundamental Theorem of Calculus (2D animation) +
67. Area as an Anti-Derivative: Compute the Area (2D animation)

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Figure 1: Constructive feedback in the Integration by Substitution maplet. Note that the user has omitted the differential (\(dx\)) in their response for \(du\). (This maplet is #68 in the Integrals: Foundations section.)

Figure 2: The Derivatives of Inverse Functions maplet, showing a function and its inverse along with tangent lines at mirror image points. (This maplet is #43 in the Derivatives: Foundations section.)
Figure 3: The Volume By Slicing maplet, showing one frame of the 3D animation and the emphasis on the setup of the definite integral. (This maplet is #90 in the Integrals: Applications section.)
Figure 4: In the Area as an Antiderivative: Derivation of the Fundamental Theorem of Calculus maplet an animation helps students understand why the FTC is true. (This maplet is #66 in the Anti-Derivatives section.)

Figure 5: Visualizing the $\varepsilon$-$\delta$ definition of the limit with the Epsilon Delta Definition of a Limit maplet. Users can control the values of $\varepsilon$ and $\delta$ using the sliders, or by entering a value in the corresponding input field. (This maplet is #28 in the Limits & Continuity section.)
Figure 6: The Convergence Tests for Series of Numbers maplet. When users select a convergence test to apply, the test appears in a separate popup window. (See the next 3 figures.) Note the graphical display of the terms (blue) and the partial sums (green). (This maplet is #108 in the Sequences and Series section.)

Figure 7: The popup window for the Ratio Test. This test is inconclusive because the terms of this series are not all positive.
Figure 8: The popup window for the Alternating Series Test. This test cannot be applied because the sequence of terms increases (in absolute value) for the first few terms.

Figure 9: The popup window for the Absolute Ratio Test. This test can be used to show the convergence of this series.
Figure 10: The **Solving Absolute Value Linear Equations** maplet supports two different ways to solve these problems. The graphical reinforcement for each step is available only after that part has been answered correctly. (This maplet is #6 in the PreCalculus: Algebra and Geometry section.)

Figure 11: The **Finding Vertical Asymptotes** maplet forces the user to work the problem algebraically and shows the graphical information only at the end of the problem. (This maplet is #11 in the PreCalculus: Algebra and Geometry section.)
Figure 12: The **Bracketing Method of Solving a Continuous Equation using Numeric Data** maplet uses visual and numeric data only – no algebra. This focuses attention on the main issue being used: the Intermediate Value Theorem. Also, the user has to decide when they can stop the iterations. (This maplet is #32 in the Limits and Continuity section.)

Figure 13: The **Tangent Plane to a Level Surface** maplet is one of the
first maplets on a multivariate calculus topic. (This maplet is #121 in the Partial Derivatives section.)

Figure 14: The Center of Mass of a Solid maplet gives students good practice setting up and evaluating triple iterated integral. (This maplet is #122 in the Multiple Integrals section.)
Figure 15: The Surface Area of Cones and Paraboloids maplet helps the user get an understanding of this often confusing topic. It's only the first maplet in this particular area of multivariate calculus. (This maplet is #124 in the Surfaces and Surface Integrals section.)