

## OVERVIEW OF MAPLETS FOR CALCULUS 1.3



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*Maplets for Calculus is a collection of 129 Maple applets for use by students and instructors in calculus and precalculus. Instructors employ the Maplets for Calculus in a variety of ways, including in-class demonstrations and lab exercises. Students find the Maplets for Calculus as an electronic study guide, even as a tutor without the tutor. In this brief document we list the mathematical topics addressed by the Maplets for Calculus. A few screenshots provide an indication of the general pedagogy, including the use of graphics. But, to get a full appreciation of the interaction provided by the Maplets for Calculus and how they can be used by instructors and students, give them a try for yourself!*

Maplets for Calculus (M4C)<sup>1</sup> is a collection of 129 Maple applets that are useful by calculus students and their instructors. The maplets in M4C provide interactive graphical user interfaces for many of the most common examples and exercises on a variety of topics in precalculus and calculus. While the M4C focused initially on single-variable topics, many of the 35 maplets new to M4C 1.3 address multivariate and vector-valued topics.

Version	Precalculus	Limits	Derivatives	Anti-Derivatives	Integrals	DifferentiaE quations	Sequences & Series	Geometry & Vectors	Partial Derivatives	Multiple Integrals	Curves & Surfaces	Games	Total
v. 1.0	2	4	11	1	13	2	2	2					37
v. 1.1	2	4	11	1	15	2	3	2					40
v. 1.2	22	8	20	4	28	4	4	2				2	94
v. 1.3	24	11	27	5	33	4	6	6	3	2	3	5	129

Table 1: Development and distribution of content of the M4C. A full listing of maplets is provided at the end of this document.

The Maplets for Calculus balance the development of both understanding and technical skills. Most maplets allow for problems to be either algorithmically-generated or user-entered.

<sup>1</sup> The current release of Maplets for Calculus, v. 1.3, was released in August 2010.. A collection of 16 free maplets is available at <http://MYmathapps.com/products/maplets-for-calculus-1-3/index.html#free> . Visit <http://MYmathapps.com/> for information about acquiring individual and site licenses to the full set of 129 Maplets for Calculus

Students find the algorithmically-generated problems very effective when developing technical skills and for building confidence as they prepare for a quiz or exam. Immediate feedback and infinite patience are two features students find particularly helpful. Students have reported that the hints and step-by-step checking of their responses is as effective as a private tutor. For example, Figure 1 shows the Integration by Substitution maplet in which the user has omitted the differential in the substitution.

There are a multitude of opportunities for instructors to use the Maplets for Calculus to enhance their courses. The ability to launch a Maplet from a webpage, PowerPoint presentation, or PDF document makes it convenient to use a Maplet for demonstrations and examples in a lecture. The extensive use of graphics – 2D, 3D, and animation – add to the impact that the Maplets can have in the classroom. Examples include the Derivatives of Inverse Functions (Figure 2), Volume by Slicing (Figure 3) and Area as an Antiderivative (Figure 4).

Others prefer to develop a project or a lab that utilizes a Maplet in a guided-discovery exercise. For example, the Epsilon Delta Definition of a Limit maplet provides a graphical and interactive environment for exploring the  $\epsilon$ - $\delta$  definition of a limit (Figure 5).

Instructors can also have their students drill on traditional topics such as Related Rates (Figure 6) and then apply these ideas to solve the classic Ladder Problem (Figure 7). Another robust collection of maplets focuses on optimization in general (Find and Classify Critical Points) and its classical applications (including separate maplets on traditional optimization applications such as minimizing the surface area of a can with a fixed volume, maximizing the enclosed area of a field divided into several pens, minimizing the surface area of a box with a fixed volume, minimizing the area of a rectangle inscribed in an ellipse, minimizing the distance from a point to a line, and minimizing the time needed to cross a river...

Some of the 35 new maplets in M4C v. 1.3 extend the coverage of precalculus and single-variable calculus topics and introduces the first Maplets for Calculus for topics from calculus of multivariate and vector-valued functions.

Many of the precalculus maplets are used by students who need a refresher on specific topics. Two examples are the Solve Absolute Value Linear Equations (Figure 10, Pre-Calc: Alg & Geom # 6) and Finding Vertical Asymptotes (Figure 11, Pre-CalcC: Alg & Geom #11). Students can use the Show buttons to see a worked example, then work through enough examples to master the skill. Graphical representations of the problem, and each step in the solution process, help students develop a better conceptual understanding while simultaneously working to improve their mechanical skills.

The authors put a lot of effort into developing the collection of four maplets for iterative methods for solving an equation. The Bracket Method for Solving ... (Figure 12, Limits & Continuity: #32 & 34) and the Bisection Method for Solving ... (Limits & Continuity: #33 and 35) are very flexible, accepting any permissible guess at each step. They also have to keep track of the error, and use this to decide when to terminate the iterations.

The maplets for multivariate topics follow the same principles that have made the univariate maplets so successful. Some of the topics addressed include: Tangent Plane to a Level Surface (Figure 13, Partial Derivatives #121), Center of Mass of a Solid (Figure 14, Multiple Integrals # 123 – beta), and Surface Area of Cones and Paraboloids (Figure 15, Surfaces and Surface Integrals. # 124).

After a listing of all 129 Maplets into 16 different categories, the remainder of this paper shows snapshots taken from several different maplets in the M4C collection.

# Maplets for Calculus v. 1.3 – Table of Contents

\* New in M4C 1.3

+ Free at MYMathApps.com, Try Before you Buy

## Precalculus: Algebra & Geometry

1. Basic 7 Functions (2D)
2. Shifting Functions (2D)
3. Reflecting Functions (2D)
4. Distance Between 2 Points \*
5. Slope of the Line Through 2 Points \*
6. Solving Absolute Value Linear Equations (2D)
7. Factor a Quadratic
8. The Shape of a Quadratic Function with a Completed Square (2D)
9. The Shape of a Quadratic, Dependence on Coefficients (2D)
10. Maximum or Minimum of a Quadratic
11. Finding Vertical Asymptotes (2D) +
12. Domain and Range Finder (2D)
13. Shifting Transcendental Functions (2D)
14. Radioactive Decay
15. Bacteria Growth
16. Solving Logarithmic Equations

## Precalculus: Trigonometry & 2D Vectors

17. Pythagorean Theorem (2D)
18. Triangle Definition of Trig Functions (2D)
19. Circle Definition of Trig Functions (2D)
20. Shifting Trigonometric Functions (2D)
21. Properties of Sine and Cosine Curves (2D)
22. Solving ASA Triangles using Trig Laws (2D)

23. Solving SAS Triangles using Trig Laws (2D)
24. 2D Vector Algebra

## Limits & Continuity

25. Left and Right Limits and Continuity, using a Graph (2D)
26. Left and Right Limits and Continuity, using a Formula
27. Left and Right Limits and Continuity, using Numeric Data
28. The Epsilon-Delta Definition of Continuity (2D)
29. Precise Definition of Limits, Linear Proofs
30. Continuity of Piecewise Defined Functions (2D)
31. Intermediate Value Theorem
32. Bracketing Method of Solving a Continuous Equation using Numeric Data (2D) \* +
33. Bisection Method of Solving a Continuous Equation using Numeric Data (2D) \*
34. Bracketing Method of Solving a Continuous Equation using a Formula (2D) \*
35. Bisection Method of Solving a Continuous Equation using a Formula (2D)

## Derivatives: Foundations

36. From Secant Slopes to Tangent Slope, using a Graph and Numeric Data (2D)
37. From Secant Slopes to Tangent Slope, using a Formula
38. Computing Tangent Lines +

39. Chain Rule
40. Derivative Drill +
41. Implicit Differentiation
42. Logarithmic Differentiation
43. Derivatives of Inverse Functions (2D) +

## Derivatives: Applications

44. Linear Approximation (2D)
45. Related Rates +
46. Related Rates: The Ladder (2D animation)
47. From Position to Velocity and Acceleration
48. L'Hospital's Rule
49. Parametric Tangent Lines
50. Identifying Graphs of First and Second Derivatives (2D) \*
51. Properties of the Graph of a Function - Terminology (2D)
52. Properties of the Graph of a Function (2D)
53. Properties of the Graph of the First Derivative (2D)
54. Properties of the Graph of the Second Derivative (2D) +
55. Finding & Classifying Critical Points \*
56. Max/Min: The Tin Can (2D animation)
57. Max/Min: Fence a Field (2D) \*
58. Max/Min: Surface Area of a Box (2D animation) \*
59. Max/Min: Rectangle in Ellipse (2D animation) +
60. Max/Min: Distance from a Point to a Line (2D) \*
61. Max/Min: Crossing a River (2D) \*
62. Linear & Quadratic Approximation

## Anti-Derivatives

- 63. Anti-Derivative Drill
- 64. Anti-Derivatives with Initial Conditions \*
- 65. From Acceleration to Velocity and Position
- 66. Area as an Anti-Derivative: Derive the Fundamental Theorem of Calculus (2D animation) +
- 67. Area as an Anti-Derivative: Compute the Area (2D animation)

## Integrals: Foundations

- 68. Integration by Substitution
- 69. Integration by Parts
- 70. Integration by Parts using Tabular Integration
- 71. Trig Integrals
- 72. Trig Substitutions +
- 73. Partial Fractions: General Decomposition
- 74. Partial Fractions: Finding Coefficients
- 75. Partial Fractions: Evaluating the Integral
- 76. Improper Integrals (2D) \*
- 77. Indefinite Integral Drill
- 78. Definite Integral Drill
- 79. Left Riemann Sums
- 80. Right Riemann Sums
- 81. Trapezoid Rule
- 82. Simpson's Rule
- 83. Left Riemann Sum Error Formula (2D) \*
- 84. Right Riemann Sum Error Formula (2D) \*
- 85. Trapezoid Rule Error Formula (2D) \*
- 86. Midpoint Rule Error Formula (2D) \*
- 87. Simpson's Rule Error Formula (2D)

## Integrals: Applications

- 88. Area of a Region (2D animation)
- 89. Average Value of a Function (2D)
- 90. Volume by Slicing (3D animation)
- 91. Volume of Revolution (3D animation)
- 92. Arc Length (2D animation)
- 93. Surface Area (3D animation) +
- 94. Center of Mass of a Bar (2D)
- 95. Centroid of a Plate (2D)
- 96. Work: Spring +
- 97. Work: Gravity
- 98. Work: Lifting with a Rope (2D)
- 99. Work: Pumping a Liquid (3D)
- 100. Fluid Force (2D)

## Differential Equations

- 101. Separable Differential Equations (2D)
- 102. Linear Differential Equations (2D)
- 103. Direction Fields (2D) +
- 104. Mixing Problems (2D)

## Sequences and Series

- 105. Sequence Drill (2D)
- 106. Geometric Series +
- 107. Telescoping Series
- 108. Series Convergence Test Drill (2D)
- 109. Computing Limits Using Maclaurin Series \*
- 110. Approximating Integrals Using Maclaurin Series \*

## Geometry & 3D Vectors

- 111. Angles of Triangles in 3D Space (3D) \*
- 112. Areas of Triangles in 3D Space (3D) \*
- 113. Angles and Areas of Triangles in 3D Space (3D) \*
- 114. Identifying Quadratic Surfaces from a Graph (3D stereo) \* +
- 115. Basic 14 Polar Curves (2D)
- 116. Polar Curve Identification (2D)

## Curves & Line Integrals

- 117. 3D Arc Length (3D) \*
- 118. Work Along a Curve (3D) \*

## Partial Derivatives

- 119. Computing Partial Derivatives \*
- 120. Maxima, Minima, and Saddle Points \*
- 121. Tangent Planes to Level Surfaces (3D) \* +

## Multiple Integrals

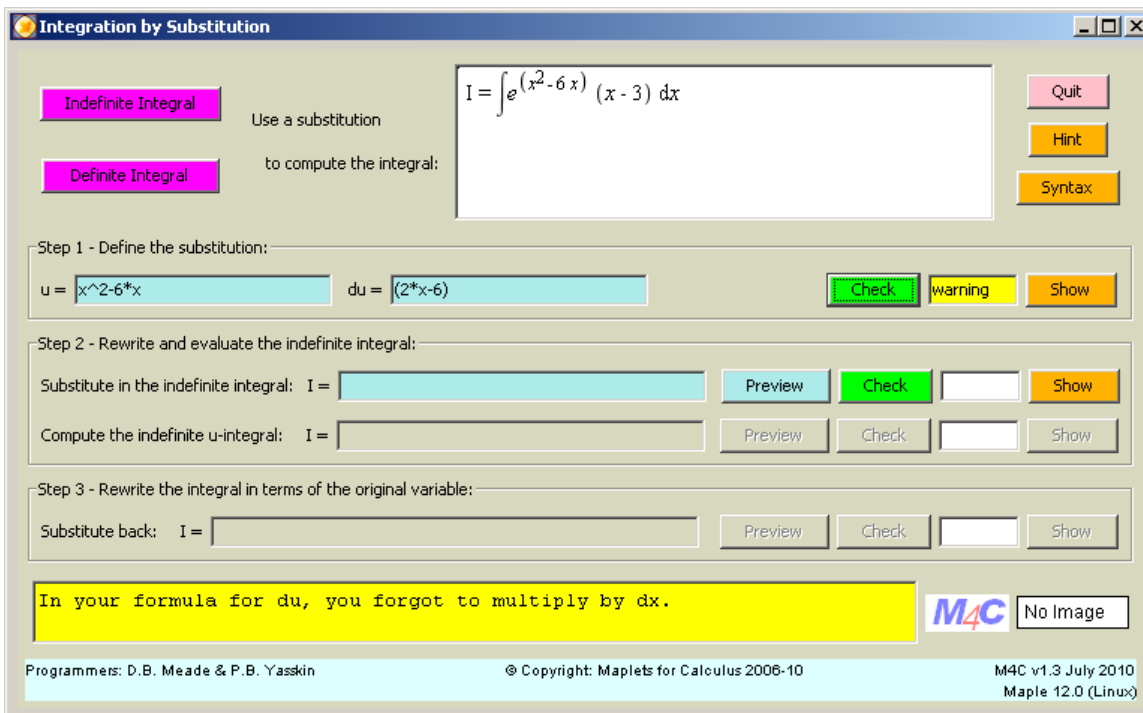
- 122. Center of Mass of a Solid (3D) \*
- 123. Limits of Integration \*

## Surfaces and Surface Integrals

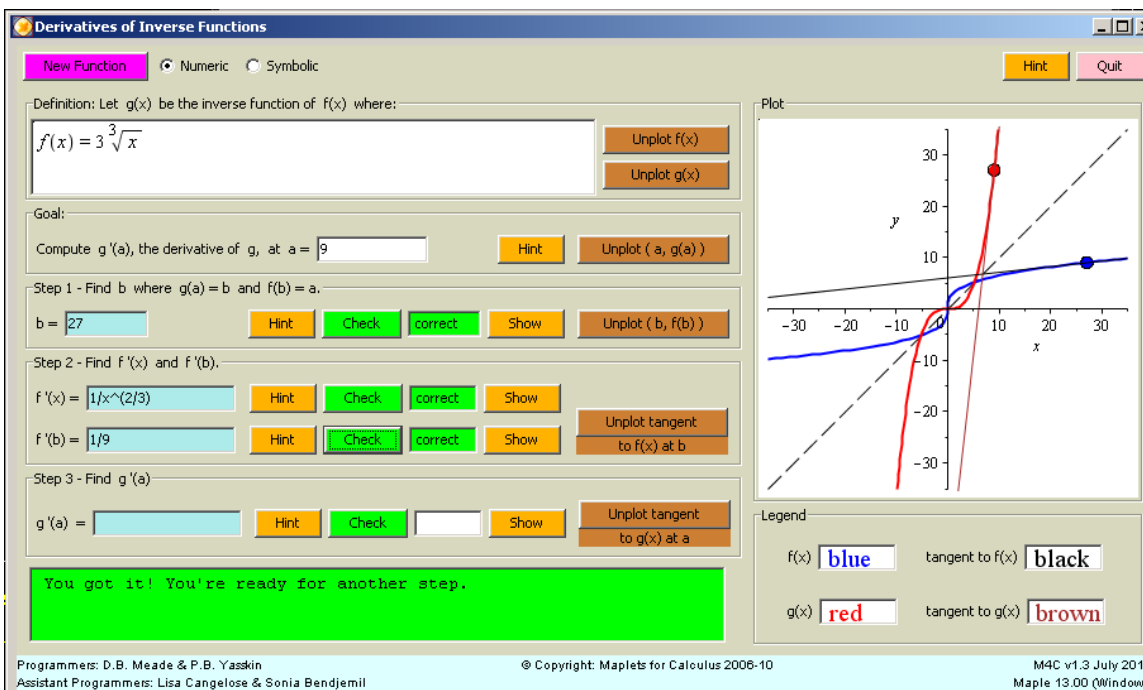
- 124. Surface Area of Cones and Paraboloids (3D) \*

## Games

- 125. Cryptogram Puzzle
- 126. The Plotting Game (2D)
- 127. Tic Tac Toe \*
- 128. 3D Virtual Reality Maze (3D) \*
- 129. Four in a Row \*



**Figure 1:** Constructive feedback in the [Integration by Substitution](#) maplet. Note that the user has omitted the differential ( $dx$ ) in their response for  $du$ . (This maplet is #68 in the Integrals: Foundations section.)



**Figure 2:** The [Derivatives of Inverse Functions](#) maplet, showing a function and its inverse along with tangent lines at mirror image points. (This maplet is #43 in the Derivatives: Foundations section.)

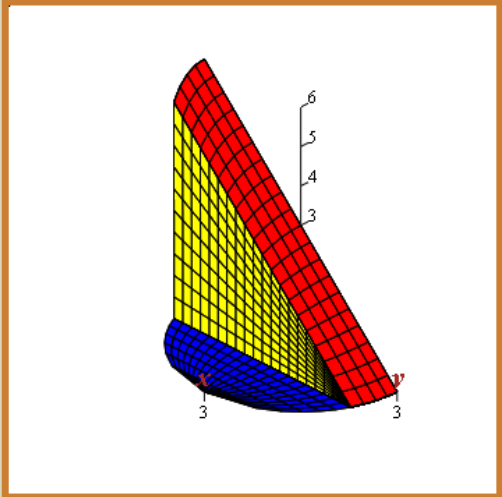
**Volume By Slicing**

New Problem or Modify or Make Your Own Problem Quit

Find the volume of the solid whose base is a semi-circle of radius 3 with the diameter edge perpendicular to the x axis, and whose cross sections perpendicular to the x direction are right isosceles triangles with a leg on the base.

Animation

Play Pause Forward Reverse



Step 1a - Enter the width of the base.  
 $W(x) = 2*(9-x^2)^{1/2}$

Step 1b - Enter the area of the cross section.  
 $A(x) = 18-2*x^2$

Step 1c - Enter the limits of integration.  
 $a = 0$        $b = 3$   
 Check correct Show

Step 2 - Compute the volume of the solid.

$$V = \int_0^3 (18 - 2x^2) dx =$$

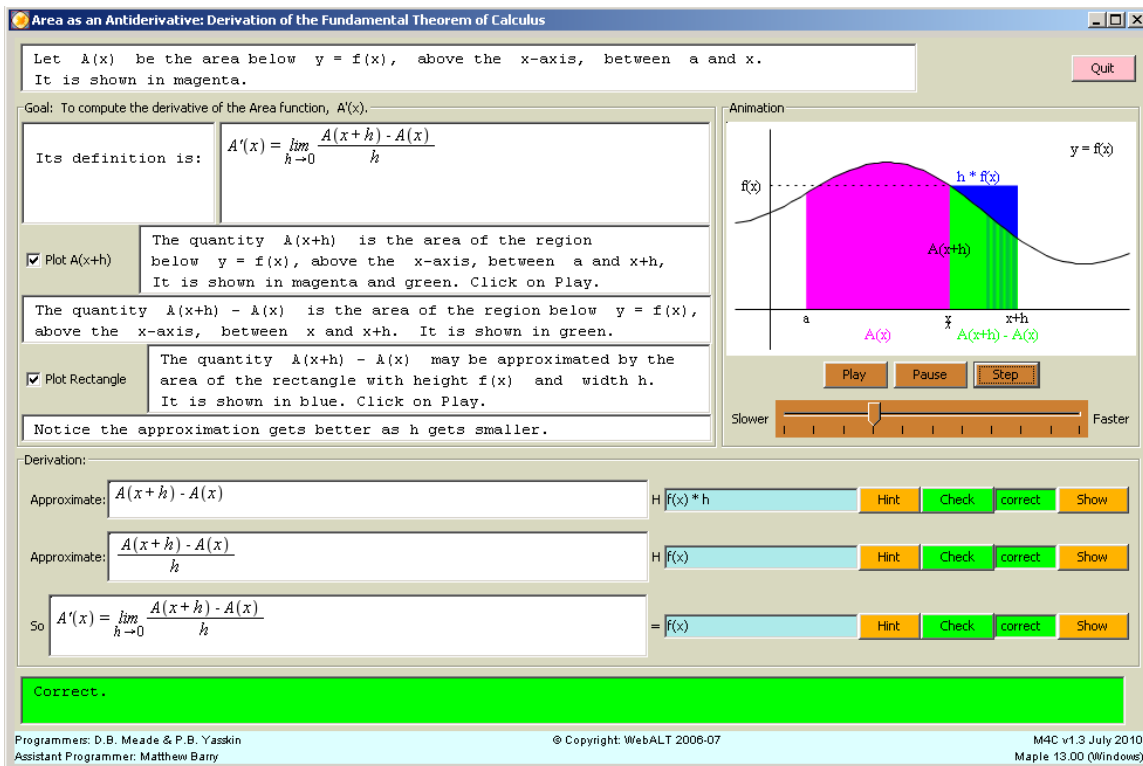
Simplify Factor Expand  
 Check Show

Oh yeah! On to the next part.

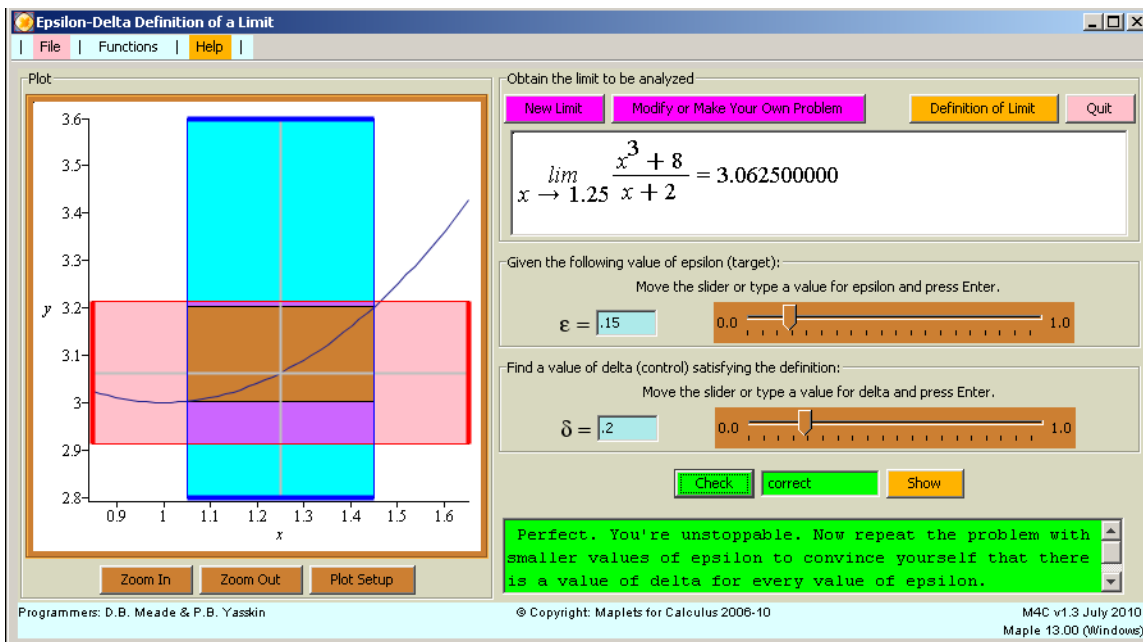
Slower  Faster

Programmers: D.B. Meade & P.B. Yasskin      © Copyright: Maplets for Calculus 2004-10      M4C v1.3 July 2010  
 Maple 13.00 (Windows)

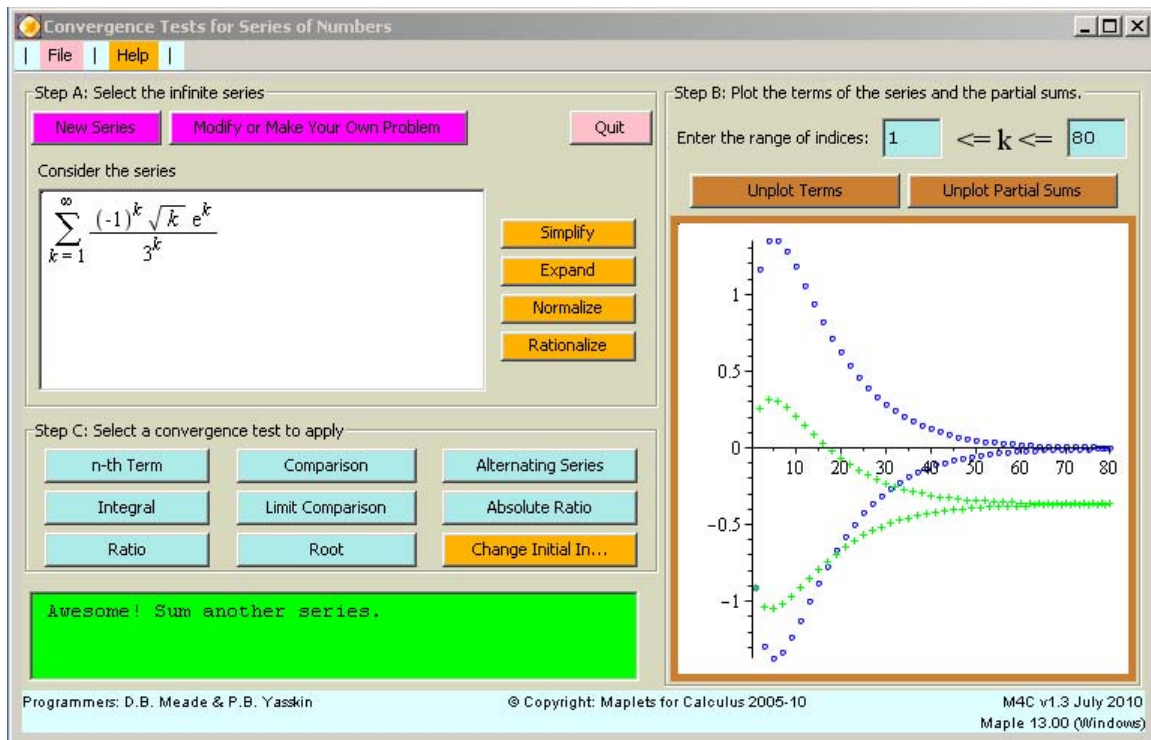
**Figure 3:** The [Volume By Slicing](#) maplet, showing one frame of the 3D animation and the emphasis on the setup of the definite integral. (This maplet is #90 in the Integrals: Applications section.)



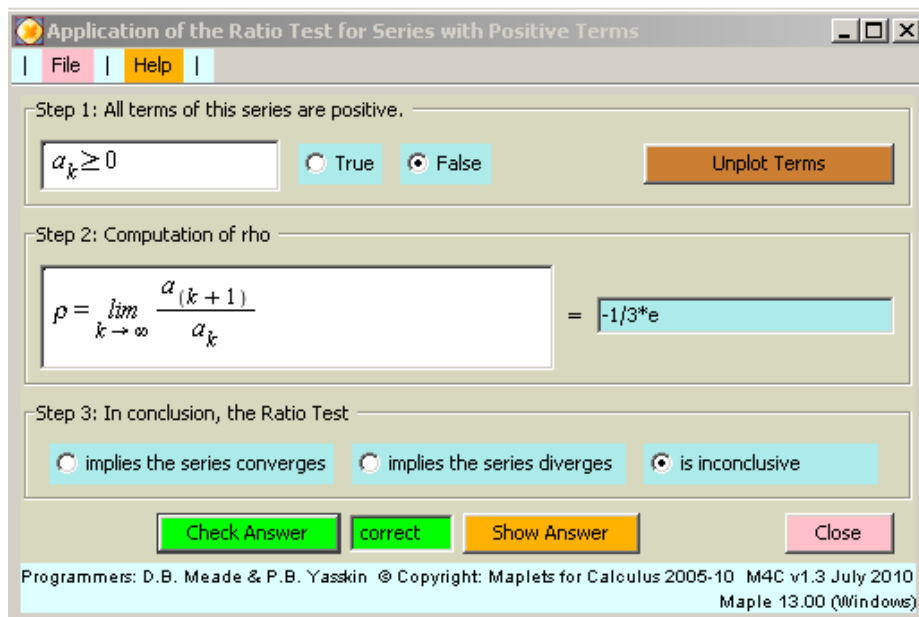
**Figure 4:** In the [Area as an Antiderivative: Derivation of the Fundamental Theorem of Calculus](#) maplet an animation helps students understand why the FTC is true. (This maplet is #66 in the Anti-Derivatives section.)



**Figure 5:** Visualizing the  $\epsilon$ - $\delta$  definition of the limit with the [Epsilon Delta Definition of a Limit](#) maplet. Users can control the values of  $\epsilon$  and  $\delta$  using the sliders, or by entering a value in the corresponding input field. (This maplet is #28 in the Limits & Continuity section.)

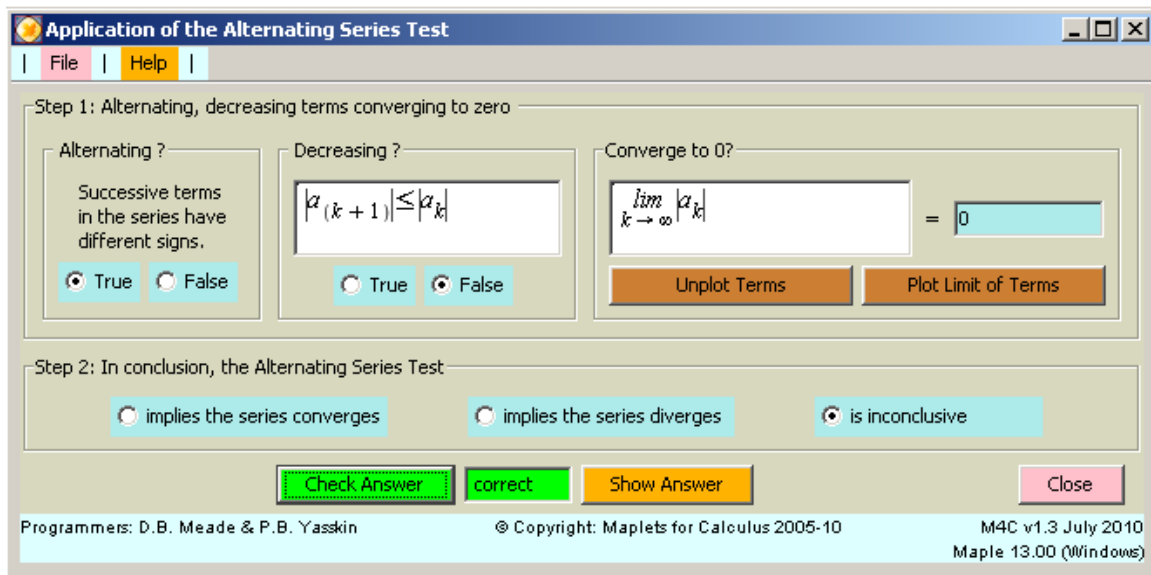


**Figure 6:** The [Convergence Tests for Series of Numbers](#) maplet. When users select a convergence test to apply, the test appears in a separate popup window. (See the next 3 figures.) Note the graphical display of the terms (blue) and the partial sums (green). (This maplet is #108 in the Sequences and Series section.)

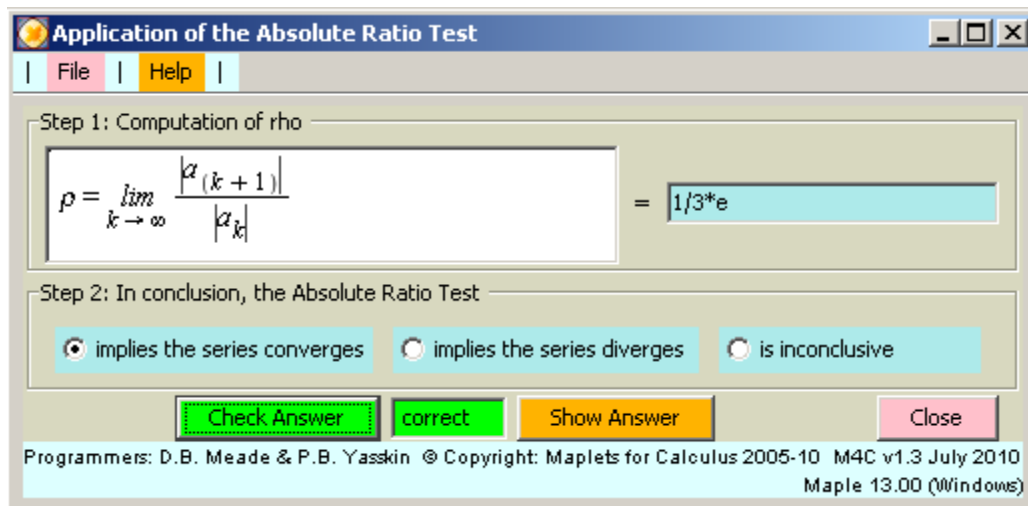


**Figure 7:** The popup window for the Ratio Test. This test is inconclusive because the terms of this series are not all positive.

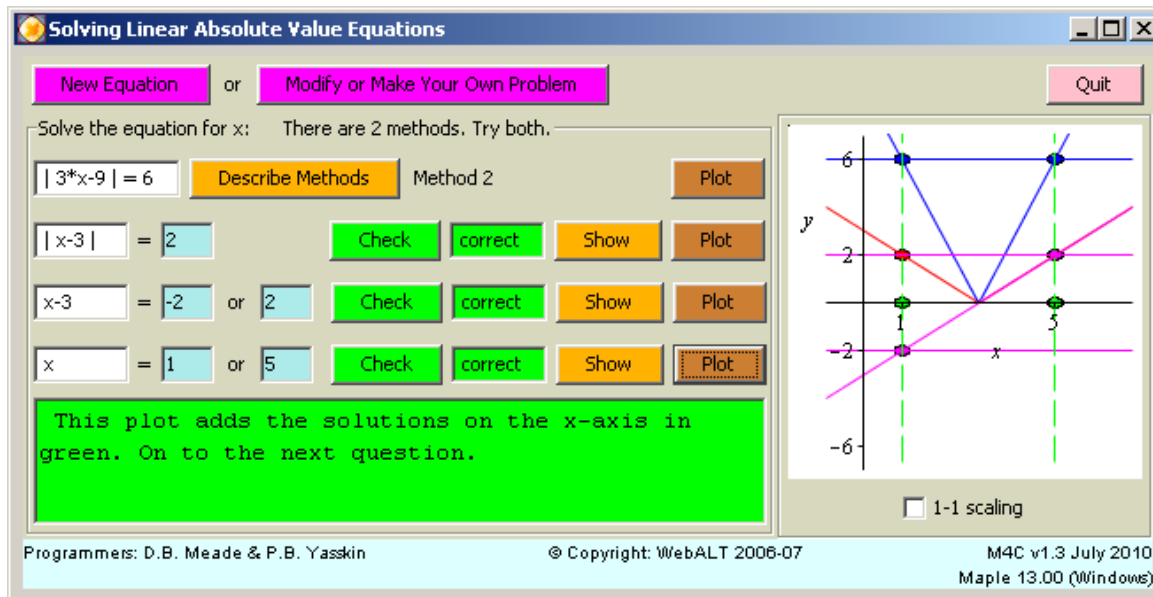




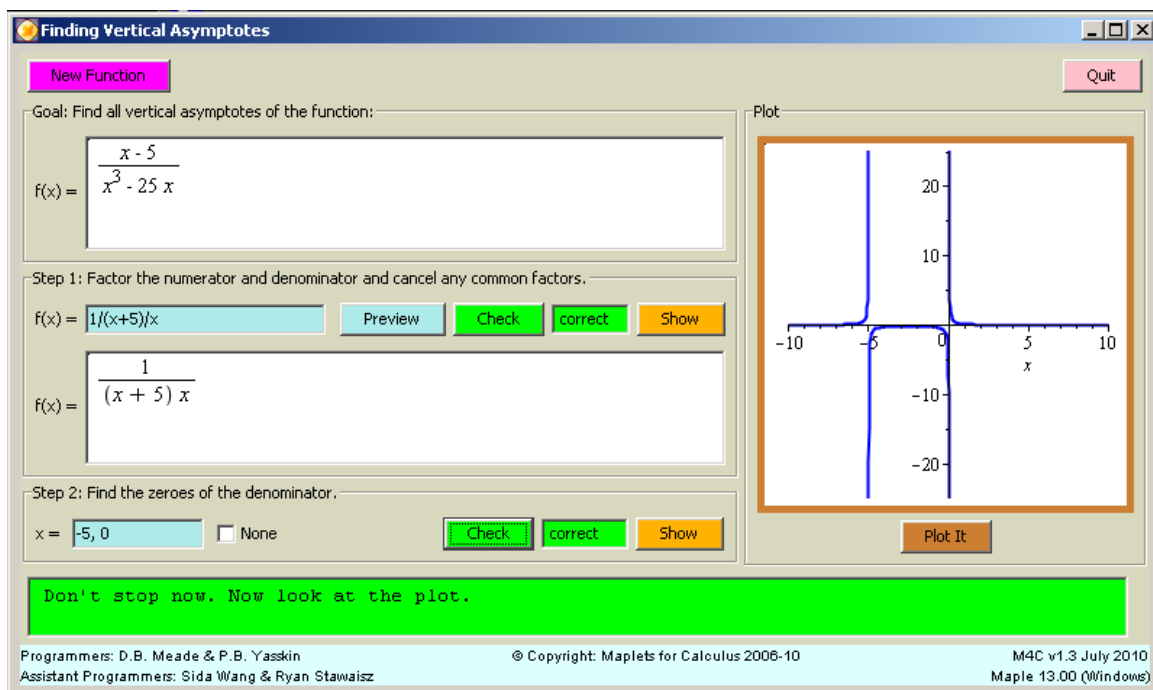
**Figure 8:** The popup window for the Alternating Series Test. This test cannot be applied because the sequence of terms increases (in absolute value) for the first few terms.



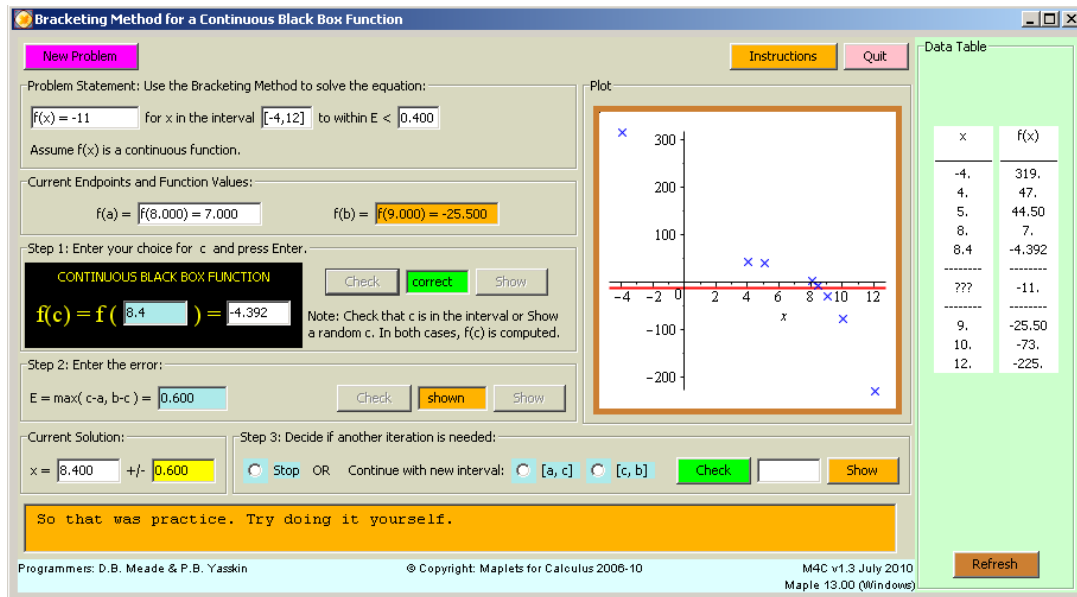
**Figure 9:** The popup window for the Absolute Ratio Test. This test can be used to show the convergence of this series.



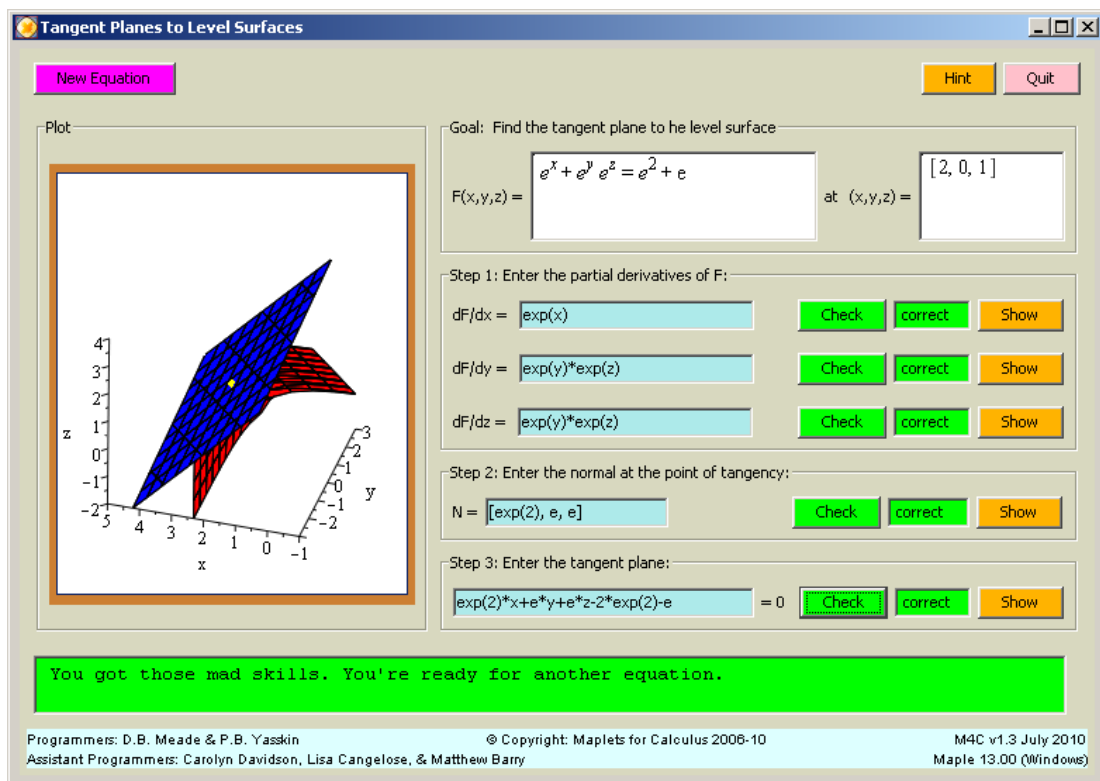
**Figure 10:** The [Solving Absolute Value Linear Equations](#) maplet supports two different ways to solve these problems. The graphical reinforcement for each step is available only after that part has been answered correctly. (This maplet is #6 in the PreCalculus: Algebra and Geometry section.)



**Figure 11:** The [Finding Vertical Asymptotes](#) maplet forces the user to work the problem algebraically and shows the graphical information only at the end of the problem. (This maplet is #11 in the PreCalculus: Algebra and Geometry section.)



**Figure 12:** The [Bracketing Method of Solving a Continuous Equation using Numeric Data](#) maplet uses visual and numeric data only – no algebra. This focuses attention on the main issue being used: the Intermediate Value Theorem. Also, the user has to decide when they can stop the iterations. (This maplet is #32 in the Limits and Continuity section.)



**Figure 13:** The [Tangent Plane to a Level Surface](#) maplet is one of the

first maplets on a multivariate calculus topic. (This maplet is #121 in the Partial Derivatives section.)

**Center of Mass of a Solid**

New Problem Quit

Find the center of mass of the solid below  $z = 3*y^2 + 2*y*x + 48$  above the region between  $x = 1 - y$  and  $x = -2 - 2*y$  for the interval  $1 \leq y \leq 5$  with density  $-2*y - 3*x + z + 10$ .

Find the total mass, M. Enter the integral and value.

M =

=  Template Preview Check Show

Find the moment about the yz-plane (integral and value):

$M_{yz} =$

=  Template Preview Check Show

Find the moment about the xz-plane (integral and value):

$M_{xz} =$

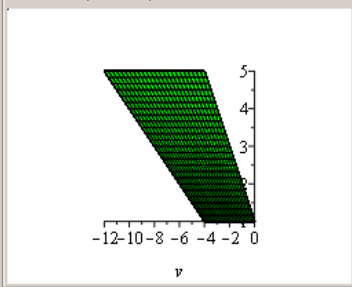
=  Template Preview Check Show

Find the moment about the xy-plane (integral and value):

$M_{xy} =$

=  Template Preview Check Show

Rotate the plot with your mouse to see the solid.



Find the x-coordinate of the center of mass:

X =  Template Check Show

Find the y-coordinate of the center of mass:

Y =  Template Check Show

Find the z-coordinate of the center of mass:

Z =  Template Check Show

Correct z center of mass!

Programmers: D.B. Meade & P.B. Yasskin © Copyright: Maplets for Calculus 2006-10 M4C v1.3 July 2010  
 Assistant Programmers: Sarah Mannen, Nick Berger, Tim Roorda, Juan Burgos and Andrew Eisterhold Maple 13.00 (Windows)

**Figure 14:** The [Center of Mass of a Solid](#) maplet gives students good practice setting up and evaluating triple iterated integral. (This maplet is #122 in the Multiple Integrals section.)

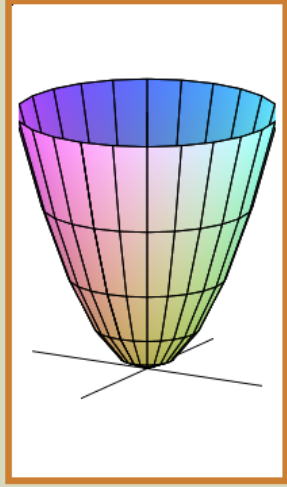
**Surface Area of Cones and Paraboloids**

New Problem Quit

Goal: Find the surface area of

the paraboloid  $z = x^2 + y^2$  parameterized by  $R(r, \theta) = [r \cos(\theta), r \sin(\theta), r^2]$  below  $z = 4$ .

Plot



Step 1: Find the maximum value of r:

$r_{\max} = 2$  Hint Check correct Show

Step 2: Find the Tangent Vectors in the r and theta directions:

$e_r = [\cos(\theta), \sin(\theta), 2r]$  Hint Check correct Show

$e_{\theta} = [-r \sin(\theta), r \cos(\theta), 0]$  Hint Check correct Show

Step 3: Find the Normal Vector and its Length:

$N = [-2r^2 \cos(\theta), -2r^2 \sin(\theta), r]$  Hint Check correct Show

$|N| = r \sqrt{4r^2 + 1}$  Hint Check correct Show

Step 4: Set up the integral for the Surface Area:

$SA = \int_0^{2\pi} \int_0^2 (r \sqrt{4r^2 + 1}) dr d\theta$  Preview Hint Check correct Show

Step 5: Compute the Surface Area:

$SA = 17/6 \sqrt{17} \pi - 1/6 \pi$  Hint Check correct Show

Power house. Integrate another function.

Programmers: D.B. Meade & P.B. Yaskin © Copyright: Maplelets for Calculus 2006-10  
 Assistant Programmers: Michael Cook, Robert Garay, & Matthew Barry M4C v1.3 July 2010  
Maple 13.00 (Windows)

**Figure 15:** The [Surface Area of Cones and Paraboloids](#) maplet helps the user get an understanding of this often confusing topic. It's only the first maplet in this particular area of multivariate calculus. (This maplet is #124 in the Surfaces and Surface Integrals section.)