Ph.D. Comprehensive Examination in Algebras, Lattices, Varieties

August 1992

Problem 0.
Prove that every complemented modular lattice is relatively complemented.

Problem 1.
Prove that in a modular lattice no element can have two distinct complements that are comparable to one another.

Problem 2.
Prove that every finitely generated distributive lattice is finite.

Problem 3.
Describe a class $\mathcal{K}$ of similar algebras such that $\mathbf{HS}\mathcal{K} \neq \mathbf{SH}\mathcal{K}$.

Problem 4.
Prove that the nonzero join irreducible elements of a complemented modular lattice are exactly the atoms of the lattice.

Problem 5.
Let $\mathcal{V}$ be a variety and let $\mathbf{F}$ be an algebra $\mathcal{V}$-freely generated by two elements. Prove that $\mathbf{F}$ has a maximal proper congruence, and that $\mathcal{V}$ contains a simple algebra.

Problem 6.
Prove that every relatively complemented lattice is congruence permutable.

Problem 7.
Prove that if $\operatorname{Con}\mathbf{A}$ is a sublattice of $\operatorname{Sub}(\mathbf{A}^2)$, then $\mathbf{A}$ is congruence permutable.

Problem 8.
Suppose that $\mathbf{L}$ is a bounded modular lattice in which 1 is the join of a finite set of atoms. Prove that $\mathbf{L}$ is a relatively complemented lattice of finite height.