Roughly speaking, abstract algebra takes as its focus sets (for example, the set of positive real numbers) equipped with some system of operations (for example, addition, multiplication, and exponentiation). Algebra is one of the core disciplines of mathematics.

After a detailed examination of several concrete algebraic systems, our class will focus on two varieties of abstract algebraic systems: groups and commutative rings. These two varieties are the ones that have found the widest range of application, both within mathematics itself and outside of mathematics. For example, one of the objectives of this two-semester course sequence is to see why angles cannot in general be trisected by straight-edge and compass, and why there is no formula like the familiar quadratic formula which will work to find the solutions to equations of degree five.

This course is the first semester of a two-semester sequence. Students planning to go on to graduate school in mathematics would find both semesters a useful foundation for that enterprise.

While I plan to give lectures, some of our time in class will be spent in discussion and working in small groups. For this reason, active personal participation is a key to the course. Your attendance and efforts will be needed during every meeting of the class.

Homework is at the heart of our course. Generally, an assignment will be due at the beginning of class every Tuesday. Homework will be collaborative. The class will be divided into small teams for the purposes of homework. Unlike the homework problems in lower division mathematics courses, our problems will require more reflection. It will usually not be possible to successfully complete a problem set the night before it is due.

Every one of you is welcome to come to my office at anytime. I will generally be in every day from 9 am until 5 pm. While I have other responsibilities, your success is my first priority. Most of the time I will be able to set aside whatever I am doing, so don’t hesitate to visit my office.

I hope you will find our course, enjoyable, informative, and useful.
How Course Grades Will be Determined

The objectives of this course can be broken down into three categories: concrete algebraic systems, groups, and commutative rings. In turn each of these categories comprises a number of objectives, some of which are essential to the application and further study of mathematics while the remaining objectives touch on more peripheral topics.

The mid-term examinations and the final will provide problems that address each objective. Your grade for the course will be determined by how well you display mastery of these problems. For each sort of problem I identify three levels of performance: master level, journeyman level, and apprentice level. The examinations will all be cumulative. The First Midterm will probably have 4 problems, the Second 8 problems (with 4 being variants of the ones occurring on the First Midterm), the Third Midterm as well as the final will probably have 12 problems. I record how well you do on each problem (an M for master level, a J for journeyman level, an A for apprentice level) on each exam. After the Final, I make a record of the highest level of performance you have made on each sort of problem and use this record to determine your course grade. If you have at some point during the semester displayed a mastery of each of the 16 sorts of problems, then your grade will be an A. If you have at some point during the semester displayed a mastery of each of the essential problems, then you will get at least a C. The grade B can be earned by displaying mastery of all the essential problems and mastery of about half of the rest of the problems. The grade D will be assigned to anyone who can master several problems but has not yet displayed a mastery of all the essential problems. In borderline cases, the higher grade will be assigned to those students who turn in their homework regularly.

Students enrolled in this course for graduate credit will be required to complete a term project due at the end of the semester.

This particular way of arriving at the course grade is unusual. It has some advantages. Each of you will get several chances to display mastery of almost all the problems. Once you have displayed mastery of a problem there is no need to do problems like it on later exams. So it can certainly happen that if you do well on the midterms you might only have to do one or two problems on the Final. (It is not unusual that a few students do not even have to take the final.) On the other hand, because earlier weak performances are not averaged in, students who come into the Final on shaky ground can still manage to get a respectable grade for the course.

This method of grading also stresses working out the problems in a completely correct way, since accumulating a lot of journeyman level performances only results in a journeyman level performance. So it pays to do one problem carefully and correctly as opposed to trying get four problems partially correctly. Finally, this method of grading allows you to see easily which parts of the course you are doing well with, and which parts deserve more attention.

The primary disadvantage of this grading scheme is that it is complicated. At any time, if you are uncertain about how you are doing in the class I would be more than glad to clarify the matter.