Name:

Midterm Examination I

Monday 18 February 2013

Problem 0

Let n, a, and b be positive integers. Suppose that n and a are relatively prime and that $n \mid ab$. Prove that $n \mid b$.

Problem 1

Let A, B, and C be sets. Let h be a function from A onto B and let g be a function from A onto C. Let $\theta_h := \{\langle a, a' \rangle \mid a, a' \in A \text{ and } h(a) = h(a')\}$. Let $\theta_g := \{\langle a, a' \rangle \mid a, a' \in A \text{ and } g(a) = g(a')\}$. Define

$$f := \{\langle h(a), g(a) \rangle \mid a \in A\}.$$

Suppose further that f is a one-to-one function from B into C.

Prove that $\theta_h = \theta_g$.

Problem 2 (**Core**)

Do each part below.

(a) Let $R = \{a + b\sqrt{5} : a, b \in \mathbb{Z}\}$. Show that R is a subring of the ring $\mathbb{R}$ of real numbers.

(b) Let R be as defined in part (a) above. Define

$$F : R \to R$$

by $F(a + b\sqrt{5}) = a - b\sqrt{5}$ for all $a, b \in \mathbb{Z}$. Prove that F is a ring homomorphism.

Problem 3 (**Core**)

Let $\mathbf{R}$ be a commutative ring. Prove that $\{r \mid r \in R \text{ and } r^n = 0 \text{ for some natural number } n\}$ is an ideal of $\mathbf{R}$.