The Theory of Computable Functions: MATH 761

Monday and Wednesday 3:55 p.m.–5:10 p.m.
LeConte 303B

Recommended Texts: Computability and Unsolvability
By Martin Davis

Computational Complexity: A Modern Approach
By Sanjeev Arora and Boaz Barak

Instructor: George McNulty
LeConte 302
Office Hours: 11:30 a.m. to 1 p.m.
Monday through Friday

Computable functions, roughly speaking, are those which can be described by computer programs. The theory of computable functions emerged in the 1930's in the work of Gödel, Turing, Post, Church, and Kleene and led directly to the development of stored program digital computers, making this theory one of the most significant applications of mathematics in the twentieth century.

This course begins with an examination of various mathematical models of computation, such as Turing machines and random access machines, on which the formal definition of computable function is based. Once the definition is in hand, the structure, extent, and limitations of the class of all computable functions becomes the focus of attention.

This leads naturally to classifying computable functions and the problems they might solve according to the demands they place on computational resources, like execution time and memory space. So a considerable part of the course will be devoted to computational complexity.

Among the applications of the resulting theory are the undecidability of formal number theory, Gödel's Incompleteness Theorem, the solution to Hilbert's Tenth Problem, which asked for an algorithm for determining those Diophantine equations which have solutions in the integers (no such algorithm exists), and the construction of highly secure encryption systems.

This course should prove useful for those who are interested in the analysis of the complexity algorithms or the application of algorithms to number theory, discrete mathematics and other branches of mathematics, as well as those who are interested in computation or in mathematical logic. The diligent student will be able to acquire a mastery of the techniques and concepts that are now in play across a wide swath of mathematics.

In the spring semester a graduate special topics course will be offered in the computational complexity of problems in discrete mathematics. MATH 761 together with this topics course would comprise a good course sequence for the purposes of the comprehensive exams.

The course grade will be determined by weekly problem sets.