Theory of Computable Functions

Problem Set on NP Completeness

Due 4 December 2017

Problem 0.

Establish the Sum Set Principle: Let $\mathbf{V}$ be a finite dimensional vector space over the two-element field. If $\bar{u} \neq \bar{v}$ where $\bar{u}, \bar{v} \in \mathbf{V}$, then $\bar{u} \odot \bar{x} \neq \bar{v} \odot \bar{x}$ for half of the vectors $\bar{x} \in \mathbf{V}$. Here $\odot$ denotes the ordinary dot product of vectors.

Problem 1.

Prove that 3COLOR is NP-complete. An instance of 3COLOR is just a finite graph $\mathbf{G}$. The task is to determine whether the vertices of $\mathbf{G}$ can be colored with three colors in just a way that no two vertices of the same color are joined by an edge.

Problem 2.

Prove that QUADEQ is NP-complete. An instance of QUADEQ is a system

$$\begin{aligned}
\sum_{i,j<n} a_{(0,i,j)} x_i x_j &= b_0 \\
\sum_{i,j<n} a_{(1,i,j)} x_i x_j &= b_1 \\
\sum_{i,j<n} a_{(2,i,j)} x_i x_j &= b_2 \\
&\vdots \\
\sum_{i,j<n} a_{(m-1,i,j)} x_i x_j &= b_{m-1}
\end{aligned}$$

where m and n are natural numbers and all the a's and b's are drawn from the two-element field. The task is to determine whether the system has a solution—that is an n-dimensional vector $(x_0, \dots, x_{n-1})$ over the two-element field which satisfies all the equations in the system.